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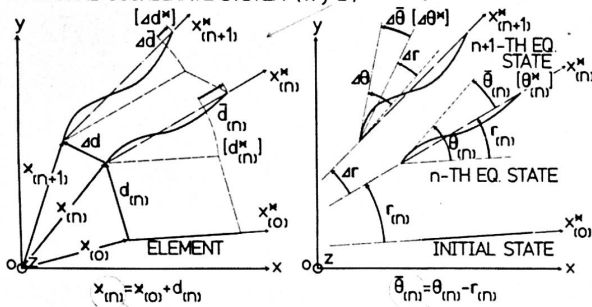
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DERIVATION OF ELEMENT EQUATION

1. COORDINATES  $x$  AND DISPLACEMENTS  $u^T = (d^T \theta^T)$  IN GLOBAL COORDINATE SYSTEM  $(x, y, z)$



2. DISPLACEMENTS  $u^{*T} = (d^{*T} \theta^{*T})$  IN LOCAL SYSTEM  $(x^*, y^*, z^*)$

$$d_{(n)}^* = \Lambda_{(n)} d_{(n)} \quad \theta_{(n)}^* = \Lambda_{(n)} \theta_{(n)}$$

$\Lambda_{(n)}$ : COORDINATE TRANSFORMATION MATRIX

3. INCREMENTAL NODAL DISPLACEMENTS IN LOCAL COORDINATE SYST. (FROM N-TH TO N+1-TH EQUILIBRIUM STATE)

$$\Delta u_{(n)}^* = u_{(n+1)}^* - u_{(n)}^* = \Delta T_{(n)}^* \begin{pmatrix} \Delta d \\ \Delta \theta \end{pmatrix} \quad T_{(n)}^* = \begin{bmatrix} \Lambda_{(n)} \\ \Lambda_{(n)} \end{bmatrix}$$

$$= \Delta T_{(n)}^* \begin{pmatrix} x \\ \theta - r \end{pmatrix}_{(n)} + T_{(n+1)}^* \begin{pmatrix} 0 \\ -\Delta r \end{pmatrix} + T_{(n+1)}^* \Delta u$$

4. FORCE-DISPLACEMENT RELATION IN LOCAL COORDINATE SYSTEM ASSUMPTION OF SMALL STRAIN

$$\Delta f^* = K^* \Delta u^* \quad K^*: \text{CONVENTIONAL LINEAR STIFFNESS MATRIX}$$

5. RELATION BETWEEN LOCAL AND GLOBAL NODAL FORCES

$$\Delta f = f_{(n+1)} - f_{(n)} = \Delta T_{(n)}^T f_{(n)}^* + T_{(n+1)}^T \Delta f^* \quad f_{(n)} = T_{(n)}^T f_{(n)}^*$$

6. FORCE-DISPLACEMENT RELATION IN GLOBAL SYSTEM [EQ. A]

$$\Delta f = \Delta T_{(n)}^T f_{(n)}^* + T_{(n+1)}^T K^* \left[ \Delta T_{(n)}^* \begin{pmatrix} x \\ \theta - r \end{pmatrix}_{(n)} + T_{(n+1)}^* \begin{pmatrix} 0 \\ -\Delta r \end{pmatrix} \right] + T_{(n+1)}^T K^* T_{(n+1)}^* \Delta u$$

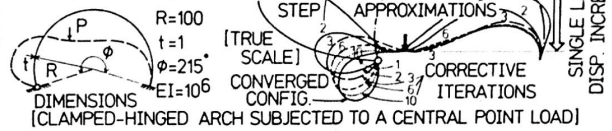
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FUNCTIONS OF UNKNOWN INCREMENTAL DISPLACEMENTS  $\Delta u$  DEVELOP. OF A TWO STEP APPROX. & CORRECT. ITERATION SOL. PROC.

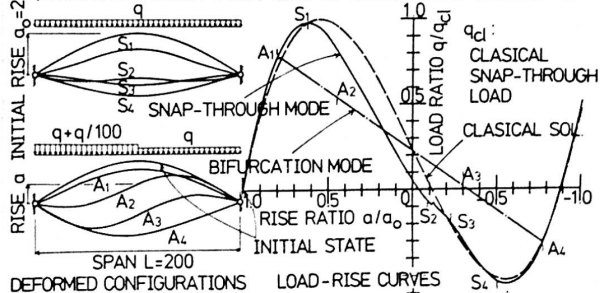
In the first two steps, EQ.A is linearized with respect to  $\Delta u$ , before the assemblage of element equations. Then in iterations, only  $\Delta u_{k+1}$  are treated as unknowns, others are estimated by using k-th approximation of  $\Delta u$ .

NUMERICAL EXAMPLES

1. CONVERGING PROCESS ILLUSTRATED BY DEFORMED CONFIGURATIONS



2. SNAP-THROUGH AND BIFURCATION ANALYSIS OF A FLAT ARCH (RISE/SPAN RATIO: 1/100) ONLY BY INCREMENTAL CALCULATION





## A COMPUTER ORIENTED FORMULATION FOR GEOMETRICALLY NONLINEAR PROBLEMS

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### FORMULATION OF THE ELEMENT EQUATION

An incremental formulation is developed here, under the assumption of large displacement but small strain. The key point of the formulation is the description of nodal locations by the coordinates themselves instead of by mere displacements. A local coordinate system ( $x^* y^* z^*$ ) is introduced which is at any time on the element itself, and the element force-displacement relation is assumed to be linear in this local system. The geometrical nonlinearity is considered only through the nonlinear transforming relations of displacement components between the space-fixed global coordinate system ( $x y z$ ) and the element-fixed local coordinate system. The transformations are evaluated rigorously without neglecting small terms, so that incremental relations satisfy equilibrium conditions after deformation as accurately as possible.

### SOLUTION PROCEDURE

To construct and solve structural equilibrium equations, an effective corrective iteration solution procedure is also originally developed. The process, outlined in the following, makes use of physical properties of each term in the derived element equation. At first, the changes of transformation matrix and rigid body rotation in the element equation,  $\Delta T$  and  $\Delta r$  respectively, are linearized with respect to incremental displacement  $\Delta u$  at the  $n$ -th equilibrium state A. And the transformation matrix after the increment,  $T(n+1)$ , is estimated at the same state A. Hence the element incremental equation is written in the following quasitangential form.

$$\Delta f = (K_f + K_u + K)_A \Delta u$$

Then summing up the element incremental equations thus approximated for overall structure and solving them, the first approximation of the  $n+1$ -th equilibrium state C is obtained. Secondly,  $\Delta T$  and  $\Delta r$  are linearized at the midpoint B between A and C, meanwhile  $T(n+1)$  is approximated at the above obtained first approximating state C. Thus the element incremental equation is written as

$$\Delta f = (K_f + K_u + K)_{B,C} \Delta u$$

Thus the second approximating solution, denoted by D, can be obtained. Then afterwards in iterations,  $\Delta T$  and  $\Delta r$  as well as  $T(n+1)$  are all evaluated by using the just preceding approximating solution, and only the incremental displacement  $\Delta u$  is treated as unknown variable. Namely, for the  $k+1$ -th approximation

$$\Delta f - h_k = K_k \Delta u$$

is used, where

$$K_k = T_k^T K^* T_k, \quad h_k = T_k^T f^*(n) + T_k^T K^* (\Delta T \begin{pmatrix} x \\ \theta - r \end{pmatrix} (n) + T_k \begin{pmatrix} 0 \\ -\Delta r \end{pmatrix})$$

and

$$\Delta T = T_k - T(n), \quad \Delta r = r_k - r(n)$$

The iteration is continued until satisfactory convergence is obtained.

### FEATURES OF THE PROPOSED METHOD

- With mere incremental calculations and without special techniques such as eigenvalue analysis, not only snap-through and limit point phenomena but also bifurcation can be pursued.
- Even for an extraordinary large increment, numerical stability exists and sufficient accuracy is obtained.
- As the consequence of the above characteristics, calculating time can be reduced considerably.