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Autor: Rockey, K.C.

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Shear Buckling of a Web Reinforced by Vertical Stiffeners and a Central Horizontal Stiffener

Flambage par cisaillement d'une âme comportant des éléments raidisseurs verticaux et un élément raidisseur horizontal en position médiane

Schubbeulen eines mit vertikalen Aussteifungen und einer zentralen horizontalen Aussteifung verstärkten Stegs

K. C. Rockey, M.Sc. (Eng.), Ph.D., A.M.I.C.E., A.M.I.Mech.E.
Engineering Department University College of Swansea

1. List of Symbols

t Thickness of webplate.
b Spacing of intermediate vertical stiffener.
d_c Clear depth of webplate.
b/d_c Effective aspect ratio.
D Flexural rigidity of unit width of plate $= \frac{E t^3}{12(1-\mu^2)}$.
E Young's modulus.
μ Poisson's ratio.
EI Flexural rigidity of stiffener.
$\gamma = \frac{EI}{D d_c} = \frac{\text{Stiffener rigidity}}{\text{Rigidity of strip of webplate equal to clear depth } d_c}$.
A Suffix V indicates a vertical stiffener.
A Suffix H indicates a horizontal stiffener.
γ_{LV} Limiting value of γ for vertical stiffeners.
γ_{LH} Limiting value of γ for horizontal stiffeners.
T Shear stress.
T_{CR} Critical shear stress.
K Critical shear stress coefficient.
K_{LV} Limiting value of K for web reinforced by vertical stiffeners only.
K_{LH} Limiting value of K for web reinforced by vertical stiffeners and a central horizontal stiffener.

2. Introduction

The use of deep plate girders with webs having a high depth to thickness ratio, has made it necessary to employ both vertical and horizontal stiffening. When a web is subjected to shear, the most effective position for a single horizontal stiffener is at mid-depth. In this paper, the influence of changes in the size of this central horizontal stiffener, and of the vertical stiffeners, upon the buckling stress of the web is examined. As a result of the investigation, new formulæ for the design of webs reinforced by both vertical stiffeners and a central horizontal stiffener have been obtained.

2.1. Background

Previous theoretical [1, 2, 3] and experimental research [4, 5] has shown that the buckling stress of a web subjected to shear can be obtained from equation (1).

$$T_{CR} = \frac{K \pi^2 E t^2}{12(1-\mu^2) d_c^2}. \tag{1}$$

The magnitude of the coefficient K will depend upon the support which the web receives from the flange members and the stiffeners. For the case of a web reinforced by vertical stiffeners of zero torsional rigidity, STEIN and FRALICH [3] have provided theoretical values of K for different values of stiffener rigidity. More recently, KLEEMAN [6] has extended STEIN and FRALICH's [3] work to allow for the effect of the torsional rigidity of stiffeners. This work showing that for open sectioned stiffeners, the effect of the torsional rigidity of the stiffener upon the buckling stress is small.

In a recent paper [7], the author presented empirical relationships between the size and spacing of vertical stiffeners and the buckling stress of the stiffened webs. These relationships were similar in form to the theoretical curves [3, 6], although as would be expected the magnitude of the values of K and γ obtained from tests on practical girders differed somewhat from the theoretical values which were obtained for idealised conditions.

It was established that for single sided stiffeners, the relationships between the buckling coefficient K, the non-dimensional parameter $\gamma \left(= \frac{EI}{Dd_c}\right)$ and the aspect ratio $\frac{b}{d_c}$ were:

$$K = K_U + A(\gamma)^{1/3}, \tag{2}$$

where K_U = critical shear stress coefficient of the unstiffened plate.

K = critical shear stress coefficient of the stiffened plate, being equal to K_{LV} for values of $\gamma \geqq \gamma_{LV}$.

A is a constant.

$$\gamma_{LV} = 21.5\left(\frac{d_c}{b}\right) - 7.5\left(\frac{b}{d_c}\right), \tag{3}$$

$$K_{LV} = 8.0 + 5.7\left(\frac{d_c}{b}\right)^2. \tag{4}$$

Unfortunately, no similar relationships, theoretical or experimental, have been obtained for the case of a web subjected to shear and reinforced by both vertical stiffeners and a central horizontal stiffener. Without such information it is obviously impossible to develop a rational design procedure for webs stiffened in this manner.

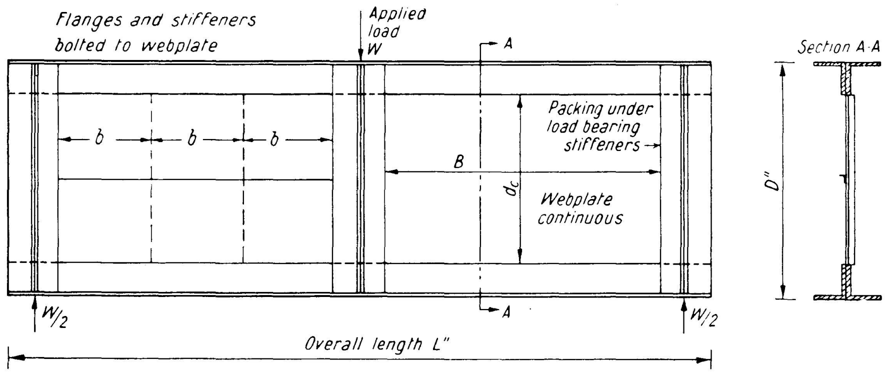
3. Experimental Investigation

3.1. Object of Investigation

The aim of the investigation was to determine the relationships which exist between the following: the size and spacing of the vertical stiffeners, the size of the horizontal stiffener and the buckling stress of the webplate. Since it was considered that only single sided stiffeners would be used when webs are reinforced by both vertical and horizontal stiffeners, it was decided to confine the investigations to the study of the behaviour of single sided stiffeners.

3.2. The Apparatus

The girders, which were constructed of a high strength aluminium alloy (H. 10. W. P.) [8], had dimensions similar to those given in fig. 1. The alloy and the dimensions of the girders were chosen to provide two essential conditions. The first being that the web was to be loaded mainly in shear and secondly,



Girder	L INS	D INS	B INS	d_c INS	b INS	Web thickness t INS
BTG/13	49.5	13	19.5	8	9.75	0.036
	49.5	13	19.5	8	4.875	0.036
	49.5	13	19.5	8	6.5	0.036
BTG/14	49.5	11	19.5	6	6.5	0.039
BTG/15	45	13.5	19.5	10	6.5	0.04
	45	13.5	19.5	10	4.875	0.04
BTG/16	49.5	14	19.5	12	4.875	0.04
	49.5	14	19.5	12	6.5	0.04

Fig. 1. Details of Girders.

that the stiffened webs could be loaded well beyond the maximum anticipated buckling stress without exceeding the 0.1% proof stress of the web material. This second requirement was absolutely necessary since a large number of tests were to be conducted on each girder. The first requirement was met by providing the girders with heavy flanges, this ensuring that the applied bending stresses were small, while the second requirement was satisfied by employing thin webs of high strength alloy.

The girders were of bolted construction since it had been found [7] that by employing this method of construction it was possible to obtain the flat webs which are essential if a well defined buckling phenomena is to be obtained.

The girders which were tested in a 100 ton amsler testing machine, were centrally loaded and supported at their ends. Dial gauges capable of recording movements of 0.0001 inch were used to measure the lateral deflections of the web. These dial gauges were held by frames which were attached to the girder in such a manner that if the girder twisted the dial gauge attachment moved with it, thus ensuring as far as possible, that the dial gauges only recorded lateral deflections of the webplate.

Electrical resistance strain gauges were also used to measure the strain distribution in the web.

3.3. Testing Procedure

The testing procedure closely followed that which has been successfully employed in the research on the buckling of webs reinforced by vertical stiffeners [7]. A set of intermediate stiffeners would be attached to the web and the zero dial gauge and electrical resistance strain gauge readings taken. The girder was then gradually loaded and the dial gauge readings, and in certain tests the strain gauge readings, taken at frequent load intervals. When it was apparent that the web had been loaded well beyond its buckling load, the girder was unloaded. Since the buckling was elastic, the web returned to its original state. It was always possible to ensure that this was so, by comparing the zero load readings obtained before and after each test. The stiffeners would then be removed and replaced by another set of stiffeners and the above procedure repeated.

4. The Analysis of the Results

The first series of tests were conducted on a web which was reinforced by stiff vertical stiffeners ($EI = 4.4\, E I_{LV}$) spaced $0.812\, d_c$ apart and by a central horizontal stiffener the size of which was varied. The procedure, as stated above, was to fit a horizontal stiffener to the web and then load the girder until it was clear that the web had buckled, then unload and replace the stiffener by another stiffener. Since the web was initially plane, well defined buckling loads were usually obtained, see fig. 2. Very occasionally however, due to deformations of the horizontal stiffener, this buckling phenomena was not so well defined.

Altogether 20 different size horizontal stiffeners were employed in this series and the results obtained are plotted in fig. 3. It will be seen that the value of the coefficient K increases with increasing values of γ until a limiting value γ_{LH}, which in this particular case is 7.5, is reached; but that there after K remains more or less constant at a value of 41.3. This constant value will be called the limiting value K_{LH}. It is of interest to note that the form of this curve is very similar to the K/γ relationships obtained from the tests on webs reinforced by vertical stiffeners only [7].

The above tests established that it was possible to obtain K/γ relationships from tests on webs reinforced by both vertical and horizontal stiffening. The next step was to determine what effect changes in the size of the vertical stiffeners would have upon the buckling stress of the web when the rigidity of the horizontal stiffener was kept constant at a value close to γ_{LH}. In particular, it was necessary to know what was the minimum rigidity the vertical

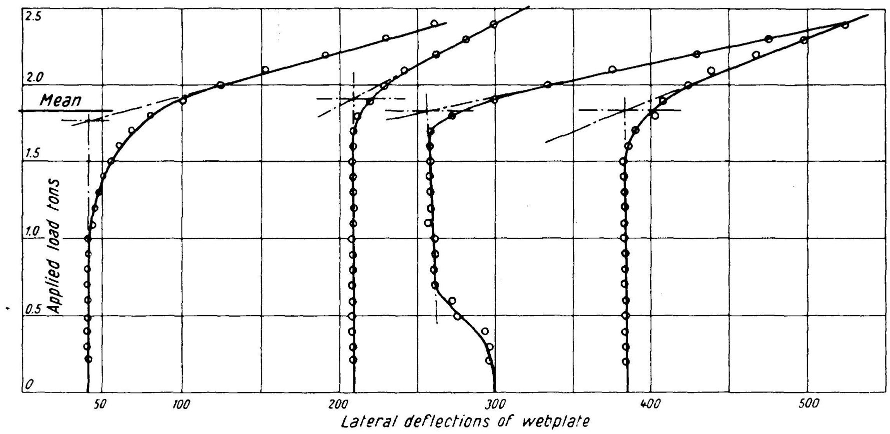


Fig. 2. Typical Load Deflection Plots.

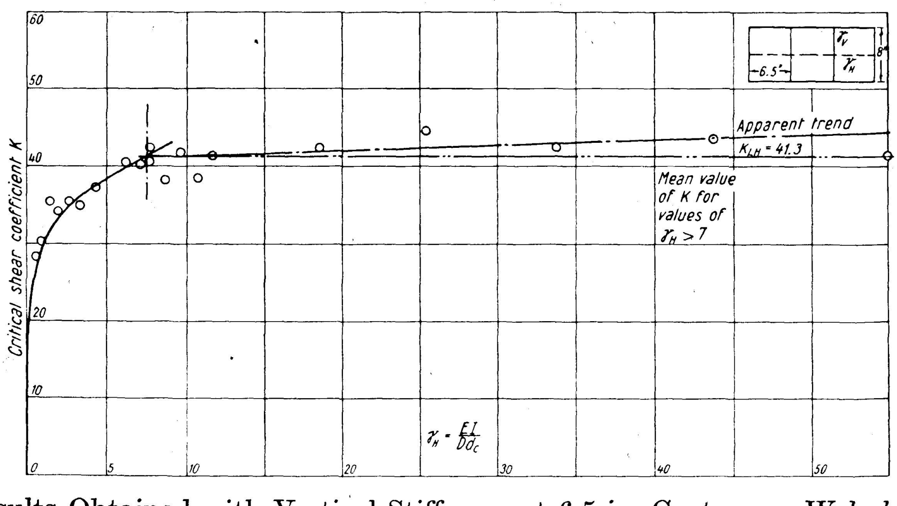


Fig. 3. Results Obtained with Vertical Stiffeners at 6.5 in. Centres on Webplate Having Clear Depth d_c of 8 Ins. γ_v of Vertical Stiffeners 93.5. Varying Size of Horizontal Stiffeners.

stiffeners could have without reducing the coefficient K below the value K_{LH}. There was no available information on this point, but in a recent paper Young and Landau [9] have proposed that the size of the vertical stiffeners should be increased wherever horizontal stiffeners were also employed.

Therefore, in the second series of tests a horizontal stiffener providing a value of γ_H equal to 1.27 times the value of γ_{LH} obtained from the first test series was fitted to the web. Twenty one different sets of vertical stiffeners were then tested. The resulting K/γ_V relationship which was obtained followed the usual pattern, see fig. 4. In this particular case the relationship between the coefficient K and the parameter γ_V is very well defined. It will be seen that the value of K_{LH} increases slightly with increases in the value of γ_V beyond the value γ_{LV}. This increase however is very slight, an increase of 700% in the value of γ_V only resulting in an 7% increase in the value of K_{LH}. This showing that only small errors are involved if it is assumed that K remains constant for values of γ_V greater than γ_{LV}.

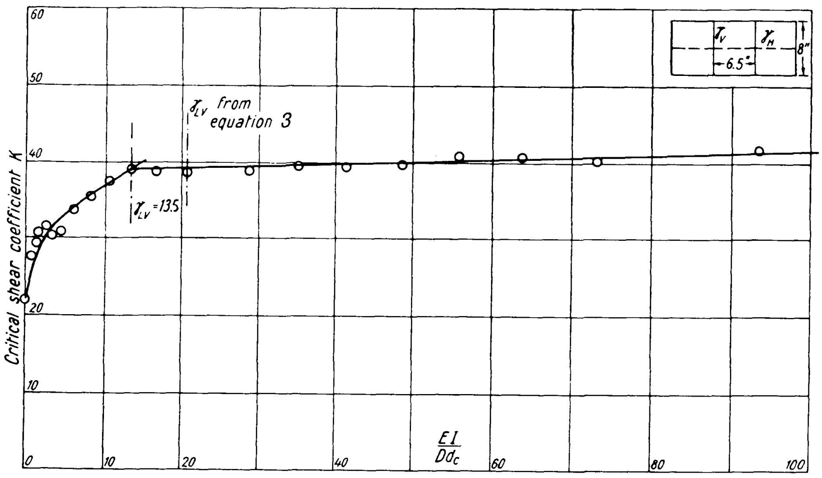


Fig. 4. Results Obtained with Vertical Stiffeners at 6.5″ Centres on Webplate Having a Clear Depth d_c of 8″$\cdot \gamma_H = 9.55$. Varying Size of Vertical Stiffeners.

The most important fact which was supplied by this test was that it was possible to decrease the rigidity of the vertical stiffeners to 66% of the value γ_{LV} obtained for the case when only vertical stiffeners are fitted without affecting significantly the buckling stress of the web. This simplified the problem considerably. If values of γ_{LH} and K_{LH} could be obtained for the case when the vertical stiffeners had a flexural rigidity $E\,I_{LV}$ it would be possible to develop a design procedure. Therefore in all the subsequent tests the vertical stiffeners were provided with a rigidity $E\,I_{LV}$ and the size of the horizontal stiffener varied. Altogether a further 102 tests were conducted and from these, values of γ_{LH} and K_{LH} obtained.

The values of γ_{LH} obtained are plotted in fig. 5, equation (5) giving the relationship between γ_{LH} and the aspect ratio b/d_c.

$$\gamma_{LH} = 11.25\,(b/d_c)^2. \tag{5}$$

The values of K_{LH} are plotted in fig. 6 and equation (6) expresses approximately the relationship between K_{LH} and the aspect ratio b/d_c. As expected, as the ratio b/d_c decreases so the experimental results tend to approach the theoretical curve for simply supported edges.

$$K_{LH} = 29 + 4.5\,(b/d_c)^{-2}. \tag{6}$$

Therefore, for a web which is reinforced by vertical stiffeners having a rigidity equal to or greater than $E\,I_{LV}$, the relationship between K and the non dimensional parameter γ_H is:

$$K = K_{LV} + A\,(\gamma_H)^{1/2}, \tag{7}$$

where $K = K_{LH}$ for values of $\gamma_H \geqq \gamma_{LH}$

$$A = a \text{ constant.}$$

Values of K_{LV}, γ_{LH} and K_{LH} being given by equations (4), (5) and (6) respectively.

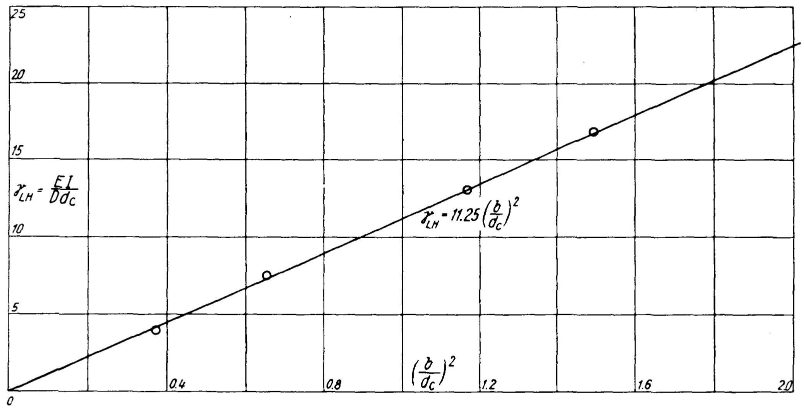


Fig. 5.

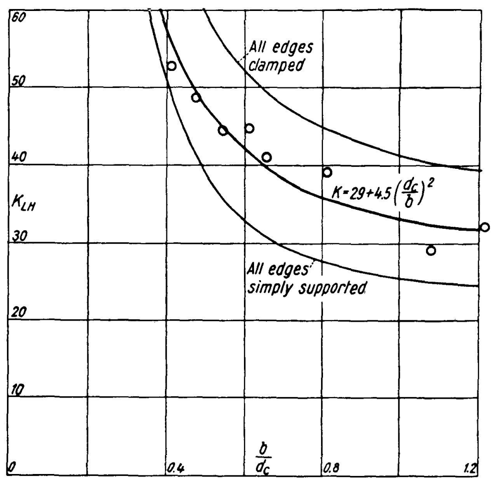


Fig. 6.

Design

In view of the fact that only limited research had been conducted on webs reinforced by both vertical stiffeners and a central horizontal stiffener, it is not surprising to find that many specifications do not incorporate rules for the design of such horizontal stiffeners. This is particularly true in the case of current British specifications [10, 11] which to date have not specified the use of horizontal stiffeners. However, the next edition of B.S. 153 will require the use of horizontal stiffeners when the web is thin relative to its depth. Unfortunately, this specification only allows for the use of a central horizontal stiffener after a horizontal stiffener has been fitted at a depth $d_c/5$ from the compression flange. Obviously, this restricted design procedure does not allow for the most economical design.

Certain European specifications [12, 13], do allow for the design of webs reinforced by a central horizontal stiffener. However, many of the rules for the design of the horizontal stiffeners have been obtained by applying the rules for the design of vertical stiffeners to the panel bounded by the vertical stiffeners and the flanges. This procedure ignores the important fact that the buckle pattern in the transverse direction is of different form to that in the longitudinal direction. STEIN and FRALICH [3] have shown that the critical stress of a panel is dependent on the wave form developed and that changes in this buckle pattern can have a significant effect upon the magnitude of the stiffener rigidity required to achieve a certain buckling stress.

One design rule, see equation (8), developed in the manner referred to above, has been plotted in fig. 7 and may be compared with the empirical law developed in section 4.

$$\gamma_H = 5.4\left(\frac{b}{d_c}\right)^2\left[2\left(\frac{b}{d_c}\right)+2.5\left(\frac{b}{d_c}\right)^2-\left(\frac{b}{d_c}\right)^3-1\right] \quad (8)$$

$$\text{valid over range } 0.5 \leqq \frac{b}{d_c} \leqq 2.0 \ [12].$$

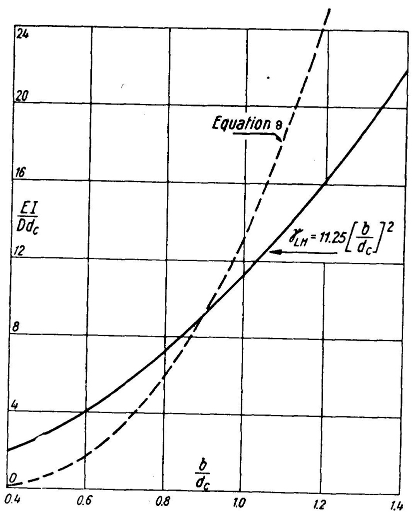


Fig. 7.

It will be seen that for values of b/d_c less than 0.9, equation (8) yields values of γ_H which are less than these obtained from equation (5). In addition, the rigidity required of the vertical stiffeners is much less than that required by equation (3), so that both the vertical and horizontal stiffening is inadequate.

Specification D 4114 also incorporates design rules for the special case of a panel which is reinforced by a central vertical stiffener and a horizontal stiffener, see fig. 8. The values of γ_V and γ_H specified in DN 4114 are plotted in fig. 8 together with the values of γ_{LV} and γ_{LH} given by equations (3) and (5)

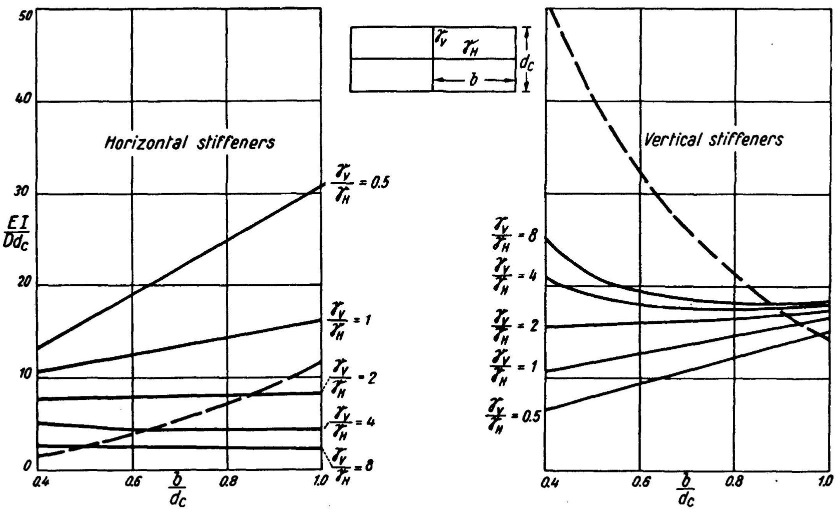


Fig. 8. --- Empirical Relationships. Equation 3 for Vertical Stiffeners. Equation 5 for Horizontal Stiffeners. —— Design Law in DN 4114[12].

respectively. It is considered that the design procedure can lead to a number of serious pitfalls. For example, no definite guidance is given as to the value of the ratio γ_V/γ_H which should be used with a given value of the aspect ratio b/d_c; although charts giving the values of the buckling coefficient K obtained with stiffeners of different rigidity are provided for the two cases when $\frac{\gamma_V}{\gamma_H}$ equals (1) and (2). This is very important, since according to the design rule, for any given value of $\frac{\gamma_V}{\gamma_H}$ of 2 or less, decreasing values of γ_V are permitted with decreasing values of b/d_c. Such a design is unrealistic. In practice, the value of $\frac{\gamma_V}{\gamma_H}$ will increase rapidly with decreasing values of the ratio b/d_c. Therefore, it is the higher values of $\frac{\gamma_V}{\gamma_H}$ which are applicable and it will be noted that for these, the rigidity required of the horizontal stiffeners is less than that required by equation (5); whilst the vertical stiffeners are generally still less stiff than those obtained from equation (3). Therefore the buckling stress of a web which is reinforced by stiffeners designed in accordance with DN 4114, will not reach the maximum possible stress which is obtainable when the stiffeners have the rigidity which is required by equations (3) and (5).

It is clear from the foregoing that the present design laws are inadequate. By using the empirical laws developed in section 4, summarised under equation (7), it is now possible to calculate the size and spacing of the vertical stiffeners and the size of the horizontal stiffeners in order to achieve a given buckling stress.

It should be noted that equation (7) gives the relationship between K and the non-dimensional parameter γ_H for the particular case where the vertical stiffeners have a rigidity equal to, or greater than $E I_{LV}$. The investigation was restricted to this case because it was not considered desirable to reduce the rigidity of the vertical stiffeners below $E I_{LV}$.

Conclusion

The new relationships between the spacing of the vertical stiffeners, the size of the central horizontal stiffener and the buckling stress of the web provide a new basis for the design of webs stiffened by vertical stiffeners and a central horizontal stiffener. This combination of vertical and horizontal stiffening can result in more economical designs than are possible when only vertical stiffeners are employed. For example, the weight of stiffening required to achieve a given buckling stress with horizontal and vertical stiffening, can be as little as 50% of the weight required when only vertical stiffeners are used.

Acknowledgement

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Summary

The paper presents the results of an experimental investigation to examine the buckling due to shear of webs reinforced by vertical stiffeners and a central horizontal stiffener. New relationships have been obtained between the size and spacing of the vertical stiffeners, the size of the horizontal stiffener and the buckling stress of the stiffened web.

Résumé

L'auteur expose les résultats d'une recherche expérimentale sur le flambage par cisaillement des âmes comportant des raidisseurs verticaux et un raidisseur horizontal en position médiane. De nouvelles relations sont établies entre les dimensions et l'écartement des raidisseurs verticaux, les dimensions du raidisseur horizontal et la contrainte de flambage de l'âme ainsi renforcée.

Zusammenfassung

Die Veröffentlichung gibt die Resultate einer experimentellen Forschung zur Untersuchung des Schubbeulens von Stegblechen, welche mit vertikalen Aussteifungen und einer horizontalen Aussteifung verstärkt sind. Es wurden neue Beziehungen zwischen der Abmessung und dem Abstand der vertikalen Aussteifungen, der Abmessung der horizontalen Aussteifung und der Beulspannung des ausgesteiften Steges gefunden.

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