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A Continuous Method for Structural Analysis of Multistory Buildings

Une méthode à milieu-continu pour l'analyse de constructions à plusieurs étages

Eine Kontinuum-Methode für die Analyse mehrstöckiger Bauten

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Introduction

Multistory buildings are generally composed of more than one type of lateral stiffening elements, namely, plane frames, coupled and simple shear walls, etc. This fact means that interaction between them must be considered in lateral load analysis. The structural analysis of such buildings consists of two methods:

1. Discrete methods, whereby a tall redundant statically indeterminate structure is obtained and for which a high speed large capacity digital computer is essential; and
2. Continuous methods whereby the discrete structure is transformed into a continuous one, by substituting horizontal connecting beams by a uniform distributed continuous lamella system, the solution is approximate but may be obtained manually or by the aid of a small desk electronic computer.

Discrete methods include the one presented by CLOUGH [2] and WEAVER [10] and WINOKUR [11] for lateral load analysis, stressing the importance of including normal strains in frame columns and coupled shear walls in the analysis. KHAN [7] and GOULD [6] also proposed approximate methods. Continuous methods have been treated by many authors [1, 3, 5, 8, 9] but there are limitations regarding the types of elements and/or normal strains in columns or shear walls. DROZDOV [4] presents a continuous method which includes the effect of normal strains and is not limited by number and type of stiffening elements. The unknowns are the normal forces in shear walls or frame columns and the solution of the problem is connected with a system of differential equations which leads to an eigenvalue problem of the order equal to the number of unknown normal forces in frame columns and coupled shear walls. For structures consisting of more than 3–4 unknown normal forces the use of a computer is indispensable due to the large amount of numerical calculations.

The object of the present paper is to provide a more simple continuous method which includes normal strain effects in frame columns and shear walls and which is not limited by number and type of stiffening elements, where a solution for practical cases may be obtained manually or by the aid of a small desk electronic computer. The unknowns of the problem are the function of structure lateral displacement $y(z)$ and a lateral load dependent function P_i for each stiffening element. These functions are derived from a system of second order linear differential equations with constant coefficients. The homogeneous part of the solution relates to an eigenvalue problem of degree equal to number of stiffening elements plus one. Because of the special form of the system the equations for the eigenvalues and eigenvector are solvable by manual procedures.

The particular solution depends on the type of loading; a trapezoidal load yields a fourth order polynomial. With the functions known all internal forces may be established.

Assumptions and Limitations

Factor 1. Floors are undeformable in their planes and have no stiffness perpendicular to these planes.

Factor 2. The stiffening elements are distributed symmetric in the plane of the structure as shown in Fig. 1.

Factor 3. The discrete structure is substituted by a continuous one replacing connecting beams by a uniform distributed lamella system.

Factor 4. In developing the method, geometric properties of e.g., the columns, beams and shear walls are vertically uniform.

This method may be derived for stepwise variations in the geometric properties by solving each zone with constant properties and adequate boundary conditions between the various zones.

Basic Differential Equation for a Coupled Shear Wall

In order to derive the method let us assume a coupled shear wall supposed to lateral load only, as shown in Fig. 2. The bending moment in each section z, is

$$M_0 = \sum_{i=1}^{2} M_i + N l, \tag{1}$$

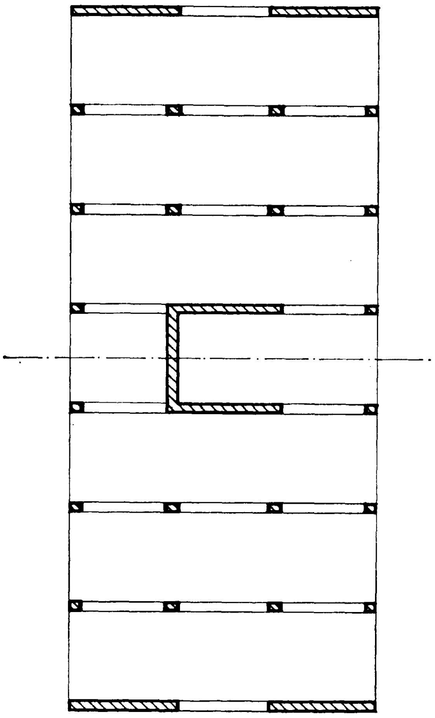

Fig. 1.

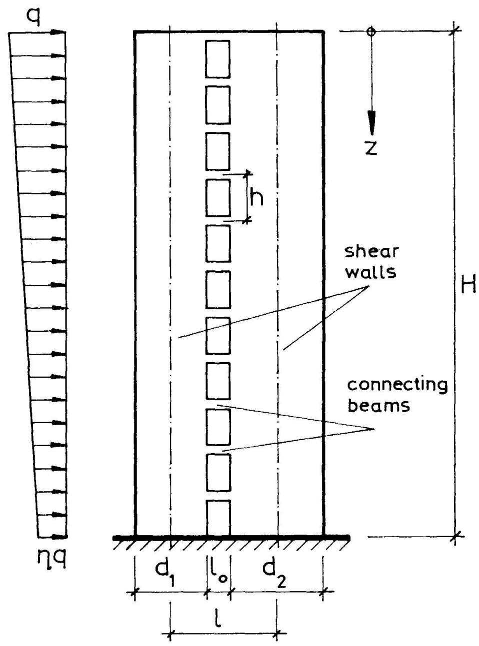

Fig. 2.

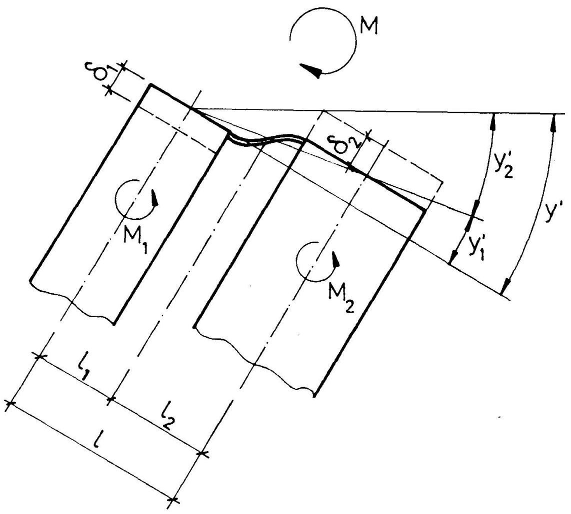

Fig. 3.

in which M_i = moment in the shear wall "i"; N = normal force in the shear wall due to lateral loads; and l = distance between shear wall center lines (see Fig. 2).

The normal force N in the shear wall is a result of bending and shear strength of the connecting beams. The increase of the normal force at story level "i" (see Fig. 3) due to deformation y_1' of the connecting beam, is

$$N_i = \frac{y_1'}{\bar{s}}, \tag{2}$$

in which

$$\bar{s} = \frac{1}{l}\left(\frac{l_0^3}{12\,E\,I_b} + \frac{l_0}{G\bar{A}_b}\right). \tag{3}$$

In Eq. (3), l_0 = clear span of the connecting beam (see Fig. 2); $\bar{A}_b$ = the effective area of the connecting beam cross section; and I_b its moment of inertia.

The equivalent of the distributed increase of the normal force per unit length of the continuous lamella system is obtained by dividing N_i by story height h. For a differential length dz the variation in the normal force, is

$$dN = \frac{y_1'}{\bar{s}} \frac{dz}{h}. \tag{4}$$

Denoting
$$s = \bar{s}\,h. \tag{5}$$

Eq. (4) becomes
$$y_1' = s\,N'. \tag{6}$$

Expressing the bending moments in the shear walls in function of y and substituting in Eq. (1), results

$$N = \frac{M_0 + y'' \sum E\,I_c}{l}, \tag{7}$$

in which, I_c = moment of inertia of shear wall cross section. Differentiating Eq. (7) and substituting in Eq. 6 yields

$$y_1' = \frac{s}{l} Q_0 + \frac{s \sum E\,I_c}{l} y''', \tag{8}$$

in which, Q_0 = overall shear force.

The angle y_2' shown in Fig. 3 may be expressed

$$y_2' = \frac{\sum |\delta_i|}{l} = \frac{1}{l}\left(\frac{1}{E\,A_1} + \frac{1}{E\,A_2}\right) \int_z^H N\,dz = \kappa \int_z^H N\,dz, \tag{9}$$

in which
$$\kappa = \frac{1}{l}\left(\frac{1}{E\,A_1} + \frac{1}{E\,A_2}\right). \tag{10}$$

Integrating Eq. (6) and substituting in Eq. (9), results

$$y_2' = \frac{\kappa}{l} \int_H^z M_0\,dz - y' \frac{k}{l} \sum E\,I_c. \tag{11}$$

The condition of compatibility yields

$$y' = y_1' + y_2'. \tag{12}$$

Substituting Eq. (8) and (11) in Eq. (12) and arranging, gives

$$K\,y''' - k\,y' = \alpha \int_H^z M_0\,dz - Q_0, \tag{13}$$

in which
$$K = \sum E I_c, \tag{14}$$
$$k = \left(\frac{\kappa}{s} \sum E I_c + \frac{l}{s}\right), \tag{15}$$
$$\alpha = \frac{\kappa}{s}. \tag{16}$$

Eq. (13) is the basic differential equation for the coupled shear wall. For other kinds of stiffening elements as frames or frames coupled with shear walls, adequated relations for κ and s coefficients are to be used. The solution of Eq. (13) is relative simple, knowing the displacement function y, all interior forces in the shear wall and connecting beam may be derived.

Common Action of Various Stiffening Elements

In structures consisting of different kinds of stiffening elements the solution gets more complicated because of the common action. The method of solution here will be the stiffness method with the displacement function y as unknown. Compatibility conditions lead to common deflection lines $y(z)$ for all stiffening elements, and the following system of differential equations for a structure with n stiffening elements is obtained.

$$K_i y''' - k_i y' - \alpha_i \int_H^z M_{0i} dz + Q_{0i} = 0, \qquad (i = 1, 2, \ldots, n), \tag{17}$$

in which M_{0i} = overall bending moment in stiffening element i; and Q_{0i} = the overall shear force in the same element.

In case of individual elements the values of M_{0i} and in consequence Q_{0i} are given data of the problem, but in the present case the manner in which the overall bending moment on the structure is distributed between the various elements is unknown. Thus the system of Eqs. (17) has $n+1$ unknowns $(y, M_{01}, M_{02}, \ldots, M_{0n})$, the values of Q_{0i} being expressed as the function of M_{0i}. The additional equation to that given by the system of Eqs. (17) is the overall equilibrium equation or its equivalent which is

$$\sum_{i=1}^{n} \int_H^z M_{0i} dz = \int_H^z M_0 dz, \tag{18}$$

in which, M_0 = overall moment acting on the structure due to lateral load. For trapezoidal lateral load as shown in Fig. 2, Eq. (18) may be written as follows:

$$\sum_{i=1}^{n} \int_H^z M_{0i} dz = -\frac{q(\eta-1)}{24H} z^4 - \frac{q}{4} z^3 + \frac{q(\eta-5)}{24} H^3. \tag{19}$$

Denoting
$$\varphi(z) = y'(z), \tag{20}$$
$$P_i = \int_H^z M_{0i} dz. \tag{21}$$

The system of differential equations may be written in the form

$$
\begin{aligned}
&P_1'' - \alpha_1 P_1 + K_1 \varphi'' - k_1 \varphi = 0,\\
&P_2'' - \alpha_2 P_2 + K_2 \varphi'' - k_2 \varphi = 0,\\
&\dots\dots\dots\dots\dots\dots\dots\dots\\
&P_n'' - \alpha_n P_n + K_n \varphi'' - k_n \varphi = 0,\\
&P_1 + P_2 + \dots + P_n = -\frac{q(\eta-1)}{24H} z^4 - \frac{q}{4} z^3 + \frac{q}{24}(\eta - 5) H^3.
\end{aligned}
\tag{22}
$$

The solution of the homogeneous part of Eqs. (22) is chosen in the form of

$$
\begin{aligned}
P_1 &= A_1 e^{rz},\\
P_2 &= A_2 e^{rz},\\
&\dots\dots\dots\\
P_n &= A_n e^{rz},\\
\varphi &= A_{n+1} e^{rz}.
\end{aligned}
\tag{23}
$$

The values of r are obtained from the characteristic equation:

$$
\begin{bmatrix}
r^2-\alpha_1 & 0\dots 0 & K_1 r^2 - k_1\\
0 & r^2-\alpha_2 \dots 0 & K_2 r^2 - k_2\\
\dots & \dots & \dots\\
0 & 0 \dots r^2 - \alpha_n & K_n r^2 - k_n\\
1 & 1 \dots 1 & 0
\end{bmatrix} = 0.
\tag{24}
$$

Denoting

$$\lambda = r^2. \tag{25}$$

Eq. (24) becomes after developing the determinant

$$\sum_{i=1}^{n} (K_i \lambda - k_i) \prod_{j \neq i}^{n} (\lambda - \alpha_j) = 0, \tag{26}$$

in which $\prod\limits_{j \neq i}^{n} (\lambda - \alpha_j) = (\lambda - \alpha_1)(\lambda - \alpha_2) \dots (\lambda - \alpha_{i-1})(\lambda - \alpha_{i+1}) \dots (\lambda - \alpha_n).$ (27)

From the nature of the problem the eigenvalues λ_i are real and positive and as consequence the values of r_i are real numbers. The numerical evaluation of the eigenvalues by the Newton-Raphson method may be obtained on a small computer. For each value r_i an eigenvector $\{A_i\}$ written as

$$
\{A_i\} = \begin{bmatrix} A_{1i}\\ A_{2i}\\ \dots\\ A_{ni}\\ A_{n+1} \end{bmatrix}
\tag{28}
$$

is obtained from the solution of the homogeneous denoted system of equations

$$\begin{bmatrix} r^2-\alpha_1 & 0\ldots 0 & K_1 r^2 - k_1 \\ 0 & r^2-\alpha_2\ldots 0 & K_2 r^2-k_2 \\ \ldots & \ldots & \ldots \\ 0 & 0\ldots r^2-\alpha_n & K_n r^2 - k_n \\ 1 & 1\ldots 1 & 0 \end{bmatrix} \begin{Bmatrix} A_{1i} \\ A_{2i} \\ \ldots \\ A_{ni} \\ A_{n+1} \end{Bmatrix} = 0. \tag{29}$$

The eigenvectors are coupled as r in Eq. (25) is squared. The positive values of $\sqrt{\lambda_i}$ are subsequently denoted by r_i.

The homogeneous solutions for all unknown functions, are

$$P_{ih} = \sum_{j=1}^{n} A_{ij}\,(C_{2j-1}\,e^{r_i z} + C_{2j}\,e^{-r_i z}), \quad (i = 1, 2, \ldots, n), \tag{30}$$

$$\varphi_h = \sum_{j=1}^{n} A_{n+1,j}\,(C_{2j-1}\,e^{r_{n+1} z} + C_{2j}\,e^{-r_{n+1} z}). \tag{31}$$

The particular solution depends on the type of loading. For a trapezoidal distributed load as shown in Fig. 2 it may be expressed in the form

$$P_{ip} = a_i z^4 + b_i z^3 + c_i z^2 + d_i z + e_i, \tag{32}$$

$$\varphi_p = a_{n+1} z^4 + b_{n+1} z^3 + c_{n+1} z^2 + d_{n+1} z + e_{n+1}. \tag{33}$$

Differentiation of Eqs. (32) and (33) and substitution into Eqs. (22) by equating the corresponding coefficients for both sides of the equations, yields

$$a_i = -\frac{\rho_i}{\sum \rho_i}\,\frac{q\,(\eta-1)}{24\,H}, \quad (i = 1, 2, \ldots, n), \tag{34}$$

$$a_{n+1} = \frac{1}{\sum \rho_i}\,\frac{q\,(\eta-1)}{24\,H}, \tag{35}$$

$$b_i = -\frac{\rho_i}{\sum \rho_i}\,\frac{q}{4}, \quad (i = 1, 2, \ldots, n), \tag{36}$$

$$b_{n+1} = \frac{1}{\sum \rho_i}\,\frac{q}{4}, \tag{37}$$

$$c_i = \frac{q\,(\eta-1)}{2\,H}\,\frac{1}{\sum \rho_i}\,\frac{K_i-\rho_i}{\alpha_i} - \frac{\rho_i}{(\sum \rho_i)^2}\sum \frac{K_i-\rho_i}{\alpha_i}, \quad (i = 1, 2, \ldots, n), \tag{38}$$

$$c_{n+1} = \frac{q\,(\eta-1)}{2\,H}\,\frac{1}{(\sum \rho_i)^2}\sum \frac{K_i-\rho_i}{\alpha_i}, \tag{39}$$

$$d_i = \frac{3\,q}{2}\,\frac{1}{\sum \rho_i}\,\frac{K_i-\rho_i}{\alpha_i} - \frac{\rho_i}{(\sum \rho_i)^2}\sum \frac{K_i-\rho_i}{\alpha_i}, \quad (i = 1, 2, \ldots, n). \tag{40}$$

$$d_{n+1} = \frac{3\,q}{2}\,\frac{1}{(\sum \rho_i)^2}\sum \frac{K_i-\rho_i}{\alpha_i}, \tag{41}$$

$$e_i = \frac{q(\eta-1)}{H}\frac{1}{(\sum \rho_i)^2}\frac{K_i-\rho_i}{\alpha_i}\sum\frac{K_i-\rho_i}{\alpha_i} - \frac{\rho_i}{(\sum\rho_i)^3}\left(\sum\frac{K_i-\rho_i}{\alpha_i}\right)^2 - \frac{q(\eta-5)}{24}\frac{\rho_i}{\sum\rho_i}H^3, \quad (i=1,2,\ldots,n), \tag{42}$$

$$e_{n+1} = \frac{q(\eta-1)}{H}\frac{1}{(\sum\rho_i)^3}\left(\sum\frac{K_i-\rho_i}{\alpha_i}\right)^2 - \frac{1}{\sum\rho_i}\frac{q(\eta-5)}{24}H^3 \tag{43}$$

in which $\rho_i = k_i/\alpha_i$ and the total solution is obtained from

$$P_i = P_{ih} + P_{ip}, \qquad (i=1,2,\ldots,n) \tag{44}$$

$$\varphi = \varphi_h + \varphi_p, \tag{45}$$

and

$$y(z) = \int \varphi(z)\,dz + C, \tag{46}$$

in which C is the constant of integration.

Boundary Conditions

As in the system of Eqs. (22) the last one is an algebraic one, the homogeneous solution given in Eqs. (30) and (31) contains only $2n$ arbitrary constants which suffices to satisfy boundary conditions for $n-1$ functions P_i and for φ. The remaining function P_i with unsatisfied boundary conditions will be obtained from the last equation in (22). At $z=0$, with moments in coupled shear walls $=0$

$$M_{0i}(0) = P_i'(0) = 0, \qquad (i=1,2,\ldots,n-1). \tag{47}$$

At $z=0$, with shear force in coupled shear walls $=0$

$$Q_{0i}(0) = P_i''(0) = 0, \quad (i=1,2,\ldots n-1). \tag{48}$$

At $z=0$, with moments in individual shear walls $=0$

$$y''(0) = 0. \tag{49}$$

At $z=H$ with lateral deflection $=0$

$$y(H) = 0. \tag{50}$$

At $z=H$ with full restraint at the support

$$y'(H) = 0. \tag{51}$$

One of these conditions will serve to derermine the constant in Eq. (46). With the solution for $P_i\,(i=1,2,\ldots,n)$ and y, known, the internal forces can be determined by means of the equations which follow

$$M_{ji} = -E I_{ji} y''(z), \tag{53}$$

$$Q_{ii} = -E I_{ji} y'''(z), \tag{54}$$

$$N_{ji} = \frac{M_{0i} - y'' \sum_{j=1}^{2} E I_{ji}}{l_i}, \tag{55}$$

$$T_{kji} = \frac{1}{h} \int_{z_k-(h/2)}^{z_k+(h/2)} N_{ji}(x)\, dx, \tag{56}$$

in which M_{ji}, Q_{ji}, N_{ji} = the bending moment, the shear force and normal force respectively in shear wall j of the stiffening element i, T_{kji} = the shear force in connecting beam at story k of the same shear wall and z_k = the ordinate of story k.

Appendix I

Coefficients s and κ for Various Types of Stiffening Elements

a) Coupled Shear Wall (see Fig. 4)

$$s = \frac{h}{l}\left(\frac{l_0^3}{12\, E J_b} + \frac{l_0}{G A_b}\right), \qquad \kappa = \frac{1}{E l}\left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}}\right).$$

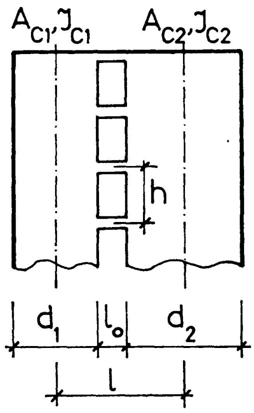

Fig. 4.

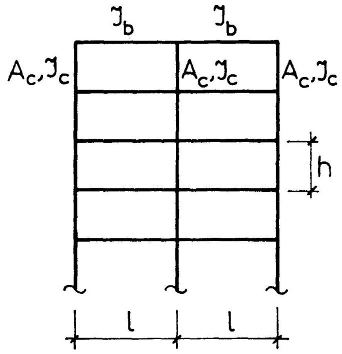

Fig. 5.

b) Frame (see Fig. 5)

$$s = \frac{h l}{12 E}\left(\frac{h}{\sum J_c} + \frac{l}{\sum J_b}\right), \qquad \kappa = \frac{1}{2 E A_c l}.$$

c) Simple Frame Wall Systems (see Fig. 6)

$$s = \frac{h}{3 l E}\left[\frac{(l_0 - u)^3 + u^3}{J_b} + \frac{h u^2}{4 J_c}\right], \qquad \kappa = \frac{1}{E l}\left(\frac{l}{A_w} + \frac{l}{A_c}\right),$$

in which

$$u = \frac{6 l_0^2 J_c}{J_b h + 12 l_0 J_c}.$$

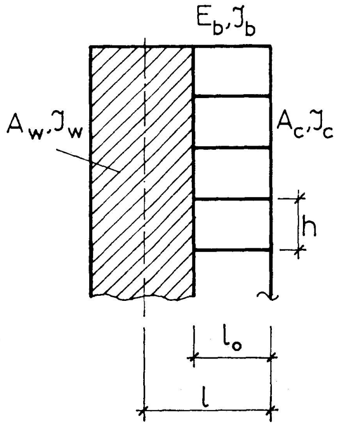

Fig. 6.

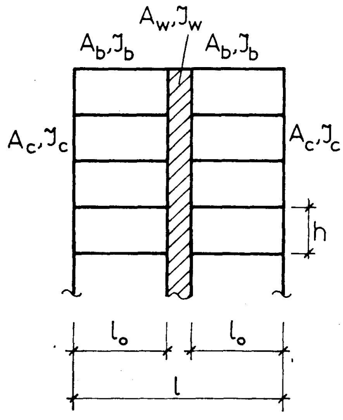

Fig. 7.

d) Double Frame Wall System (see Fig. 7)

$$s = \frac{h}{3\,l\,E}\left(\frac{(l_0-u)^3+u^3}{J_b}+\frac{h\,u^2}{4\,J_c}\right), \qquad \kappa = \frac{1}{2\,E\,A_c\,l},$$

in which

$$u = \frac{6\,l_0^2\,J_c}{J_b\,h+12\,l_0\,J_c}.$$

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Summary

This paper gives a method of lateral load analysis of symmetric multistory structures consisting of frames, simple or connected shear walls. The method is based on the continuum concept in which the frames and connecting beams are replaced by an equivalent continuous media. The number of stiffening elements is not limited and the effect of normal strains in shear walls is included in the analysis. Practical solutions may be obtained by add of a small desk electronic computer.

Résumé

Ce travail présente une méthode d'analyse pour charges latérales à des charpentes symétriques à plusieurs étages qui consistent en des cadres à plans simples ou combinés. La méthode est basée sur l'idée du milieu-continu, suivant laquelle les cadres et poutres reliantes sont remplacés par des milieux équivalents et continus. Le nombre des éléments d'entretoisement n'est pas limité; l'influence des déformations simples des plaques est comprise dans l'étude. Des résultats pratiques peuvent être obtenus aisément moyennant l'aide d'une calculatrice électronique sur pupitre.

Zusammenfassung

Die Arbeit berichtet über eine Methode der seitlichen Belastungsanalyse an symmetrischen, mehrstöckigen Bauten, bestehend aus Rahmen, aus einfachen oder zusammenwirkenden Scheiben. Die Methode stützt sich auf das Kontinuum-Konzept, bei welchem die Rahmen und verbindenden Träger durch gleichwertige, kontinuierliche Medien ersetzt sind. Die Zahl der Versteifungselemente ist nicht begrenzt, und der Einfluss normaler Scheibenverformungen wird in der Untersuchung beachtet. Praktische Lösungen lassen sich unter Hinzuziehung eines kleinen elektronischen Tischrechners erzielen.

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