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Autor: Yabuki, Tetsuya / Kuranishi, Shigeru

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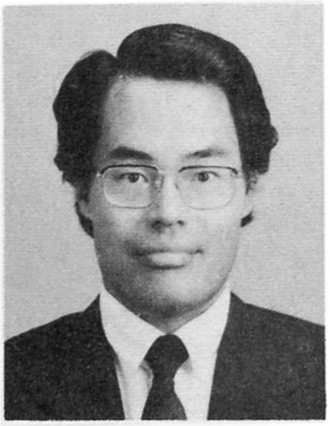
Ultimate Strength Design of Steel Arch Bridge Structures

Vérification à la ruine de ponts arc en acier

Tragfähigkeitsnachweis von Stahlbogenbrücken

Tetsuya YABUKI
Assoc. Professor
Univ. of the Ryukyus
Okinawa, Japan

Shigeru KURANISHI
Professor
Tohoku University
Sendai, Japan



Tetsuya Yabuki, born 1944 got his degree of Doctor of Engineering at Tohoku University, Sendai, Japan. For about ten year he was involved in the load carrying capacity problems specialy on steel arch structures. Tetsuya Yabuki now is researching the ultimate strength behavior and design aid for curved box girders with high depth.

Shigeru Kuranishi, born 1930 got his degree of Doctor of Engineering at Tokyo University, Tokyo, Japan. He is researching on the dynamic load carrying capacities of steel structural members under axial compression.

SUMMARY

This paper presents design criteria of two-hinged, parabolic, steel arches based on the ultimate limit state design concept, as a development in the ultimate strength design of steel arch bridge structures. The design criteria on the planar ultimate strength are formulated for arch ribs and for stiffened arch bridge structures with a deck type girder. However, the local failure and/or local buckling of the arch ribs are not considered. Several important findings are presented in the derivation process of these design formulas.

RÉSUMÉ

Cet article présente des critères de dimensionnement basés sur l'état ultime de ruine pour des ponts arc paraboliques en acier avec rotules aux appuis. Les formules de dimensionnement à la ruine d'une section sont développées pour le cas des arcs seuls et le cas d'un pont arc contreventé avec tablier en poutre continue. La rupture localisée ou le voilement ne sont pas pris en compte. Plusieurs constatations importantes qui ont pu être faites pendant le développement des formules de dimensionnement sont rapportées.

ZUSAMMENFASSUNG

In diesem Beitrag werden Bemessungskriterien für parabolische Zweigelenk-Bogenbrücken aus Stahl vorgestellt, die sich auf das Konzept des Bruchgrenzzustandes stützen. Die Bemessungsformeln für den Tragfähigkeitsnachweis von Bemessungsquerschnitten sind für Bogenrippen und für ausgesteifte Bogenbrücken mit einem Fahrbahnträger im einzelnen dargelegt. Lokales Versagen und Beulen bleiben jedoch unberücksichtigt. Hiezu werden mehrere wichtige Erkenntnisse, die sich bei der Herleitung dieser Bemessungsformeln ergeben haben, vorgestellt.

1. INTRODUCTION

In the last decade, the design practice for various types of steel structures have been changed to limit state design rules to obtain more rational designs. As for steel arch structures the classical critical instability is often regarded as the chief design criterion[2]. However, this is by no means the ultimate load which the arches with practical proportionings can carry. The actual carrying capacity of the arches with practical proportionings under practical loadings (i.e., unsymmetrical loadings) is considerably less than the classical bifurcation buckling[5],[6].

This paper concerns itself with a development in the ultimate strength design of two-hinged steel arch bridge structures based on their over-all load carrying capacities (-no local failures-) analyzed by the numerical approach. It is based on the finite element technique and the modified Newton-Raphson procedure using the incremental load method and the tangent modulus method. It considered the influence of finite deformations, spread of yielding zones in cross sections and along the longitudinal axis, welding residual stresses and unloading due to strain reversal. The verification of this analysis has been carried out in Ref.[3] by comparing with the experimental results. The interaction of bending moment and axial thrust on the ultimate strength of the arch ribs is a complex phenomenon which is extremely important in their practical situations. This phenomenon is considered and the design criteria are specified by bending moment and axial thrust at a quarter point of the arch ribs, which are all calculated by the first order elastic analysis.
The design criteria include the ultimate strength formulas for the arch ribs and for the stiffened arch structures with a deck type girder.

2. DESIGN FORMULATION FOR THE ARCH RIBS

2.1 General Behavior

The arch configuration adopted herein is shown in Fig.1. The range of the structural parameters used in the analysis are as follows:

$$\lambda = 100\sim300;\ h/L = 0.1\sim0.3;\ \sigma_Y = 240\sim480\ \mathrm{N/mm^2};\ E = 210000\ \mathrm{N/mm^2};$$

in which λ = slenderness ratio of an arch rib given by the ratio of the curvilinear length of arch axis to the radius of gyration of the cross section r_x, σ_Y = yield stress level, and E = Young's modulus. The distribution pattern of load q is given by the loading parameter r - which is considered to vary from 0 to 0.99 herein - as shown in Fig.1. The influence of a concentrated load is considered, corresponding to the line load given by Japanese Specifications of Highway Bridges[1]. Therefore, the proposed design formula is valid in the above-mentioned domains.

Useful findings that are brought out by the numerous numerical analysis [3], [5],[6] within the above-mentioned domains with respect to general behavior of two-hinged steel arch bridge structures loaded to the ultimate state in their plane are summarized as follows:

1) Steel arch bridge structures under the practical loadings become instable (i.e., load carrying capacities) before the loads reach the critical values that produce the elastic instabilities or their plastic hinge formulations.
2) The unsymmerical distributed loading pattern as shown in Fig.1 gives a lower load carrying capacity for the arch than the symmetrical loading.
3) The load carrying capacity of the steel arch ribs depends chiefly on the slenderness ratio, the yield stress level of the material, the rise/span ratio, the unsymmetry of the distributed loads, and the concentrated load placed on a quarter point of the arch rib.
4) The effect of the variation of cross sectional proportions on the ultimate

load intensity nondimensionalyzed by the full plastic load q_p is not significant, if the column slenderness ratio parameter $\bar{\lambda}$ for arch ribs is equal each other. $\bar{\lambda}$ and q_p are given by [3]

$$\bar{\lambda} = \lambda\sqrt{\sigma_Y/E}\ /\ \pi \quad \text{(1)}$$

$$q_p = A_a\sigma_Y/\sqrt{(\frac{n-2}{2})^2 + [\sum_{i-1}^{n} \frac{5Ll_1l_2}{8h}(l_1^2 + 3l_1l_2 + l_2^2)]^2} \quad \text{(2)}$$

in which n = number of the nodes of an arch rib, $l_1 = (i-1)/n$, $l_2 = 1-l_1$, i = order number of the nodes of an arch rib (Fig.1).

5) The influences of the slenderness ratio and the yield stress level on the load carrying capacity are collectively evaluated by the so-called column slenderness ratio parameter $\bar{\lambda}$.

6) The ultimate load intensity increases in proportion to the cross sectional area.

2.2 Expressions of Design Criteria

Actual design loading on the arch bridge structures usually produce both axial compression and bending moment on a general cross section of the arch rib. These internal forces give considerable effects on the geometrical and material nonlinearlities of the arches and eventualy on the load carrying capacities. Accordingly, it is reasonable and rational that the design formulas are expressed by the critical axial thrust and bending moment. From the viewpoint of design practice, it is desirable that the design formulas of arch ribs are expressed by the critical cross sectional forces calculated by the 1st order elastic analysis, even if mathematical expressions of the formulas become slightly complicated. The calculated, critical, external forces for the ultimate strength of the arches are reviewed. Fig.2 shows typical examples of the interaction curves between the nondimensional maximum axial force N_{max}^{1st}/N_Y and bending moment M_{max}^{1st}/M_Y at a typical point (i.e., a quarter point) of arch ribs. Here, these maximum forces are calculated by the 1st order elastic analysis for the arch under the ultimate load, and N_Y, M_Y, are the squash axial force and yield bending moment, respectively. From these results, it can be observed that the interaction curves may describe a quadratic line when the maximum axial force is greater than a certain critical value, while the interaction curves show linear functional relatioship when the maximum axial force is less than the critical value. This critical value is defined as n_{cr} in this paper. By considering these examination results, the following design formulas are proposed herein:

$$f = 1 \quad \text{(3a)}$$

$$a\left(\frac{M_{max}^{1st}}{M_Y}\right)^2 + b\left(\frac{M_{max}^{1st}}{M_Y}\right) + c\left(\frac{N_{max}^{1st}}{N_Y}\right) = f\ ;\ \text{for}\ \frac{N_{max}^{1st}}{N_Y} > n_{cr} \quad \text{(3b)}$$

$$\alpha\left(\frac{M_{max}}{M_Y}\right) + \beta\left(\frac{N_{max}}{N_Y}\right) = f\ ;\ \text{for}\ \frac{N_{max}}{N_Y} \leq n_{cr} \quad \text{(3c)}$$

where a, b, c, α, and β are coefficients depending on $\bar{\lambda}$ and h/L. These coefficients and λ, h/L can be expressed by functional formulas as follows:

$$a = 2.509-1.689\bar{\lambda}\ ;\ b= -1.213+1.605\lambda-0.135\bar{\lambda}^2\ ;$$

$$c = (1.824-0.014\bar{\lambda}+0.376\bar{\lambda}^2)(0.82+1.2h/L);\quad \alpha= 1/m_p\ ;\ \beta = (m_p-m_{cr})/(m_p n_{cr});$$

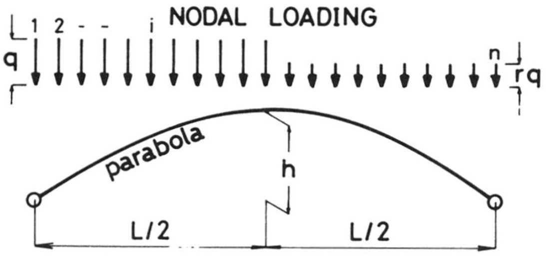


Fig.1 Arch Geometry and Loading

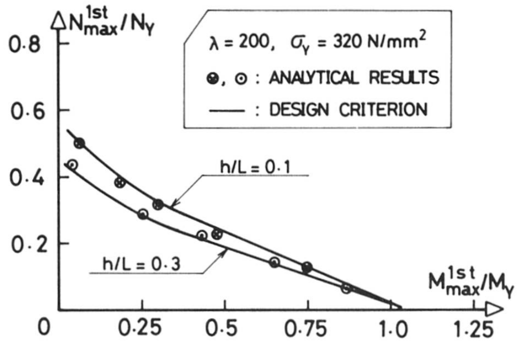


Fig.2 Relationship Between Maximum Thrust & Bending Moment

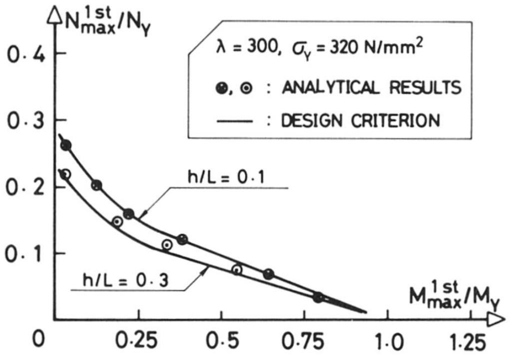


Fig.3 Accuracy of the Design Formula (for λ = 200)

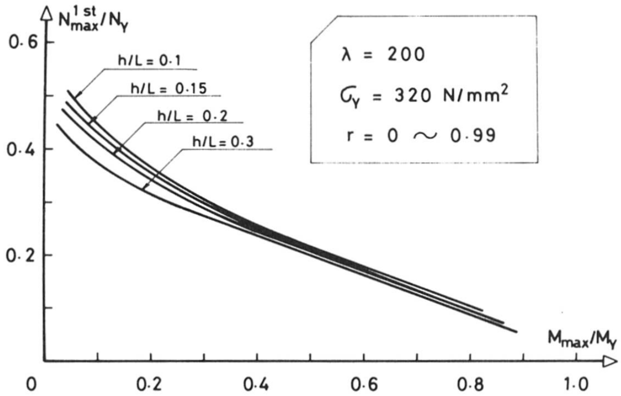


Fig.4 Accuracy of the Design Formula (for λ = 300)

$m_p = 1.172-0.0469\bar{\lambda}$; $n_{cr} = (1-bm_{cr}-am_{cr}^2)/c$;

$m_{cr} = m_p-\sqrt{(am_p^2+bm_p-1)/a}$ for $(am_p^2+bm_p-1)\geq 0$; $m_{cr} = m_p$ for $(am_p^2+bm_p-1)<0$.

For demonstration of the accuracy of the ultimate strength design formula - Eqs.3 - proporsed herein, comparison with the exact values calculated by the ultimate strength analysis are made for various cases within the afore-said domains of the structural parameters. Some investigated results are also shown in Figs.3 and 4. The solid lines in these figures illustrate the interaction curves between N_{max}^{1st} and M_{max}^{1st} given by the proporsed design formula and ⊗ and ⊙ marks show the analyzed results for h/L = 0.1 and 0.3 respectively. It will be seen from these figures that the results predicted by the proposed design formula and the analyzed ones agree fairly well. Table 1 shows the availability of the design criteria for the cases of the additional concentrated load Q_c. Here, q_{max} is the ultimate load intensity of an arch subjected to distributed load, q_{max}^* is the distributed load intensity in the ultimate state of the arch under the distributed load and the concentrated load Q_c, f(DESIGN) is Eq.3a and f(ANALYSIS) is the f-value calculated by substituting N_{max}^{1st} and M_{max}^{1st} values for the arch under q_{max}^* and Q_c into Eqs.3b and 3c. From the results in Table 1, it can be seen that, with the investigated range, the proposed design formula always provides slightly cocervative evaluation - f(ANALYSIS) 1 - for the ultimate strength of the arch ribs. Therefore, the design formula is also applicable to the additional, concentrated, loading cases.

2.3 Incorporation of the Safety Factors

In this paper, a standard of the safety factor or load factor that should be considered in designing steel arch bridge structures is not examined. However, noting that the proposed criteria are established on the values calculated by the 1st order elastic analysis, it could be permissible to determine the allowable sectional forces N_{all}, M_{all} by dividing those critical forces N_{max}^{1st}, M_{max}^{1st}, in Eqs.3 by conventional safety factor SF as follows:

$$N_{all} = N_{max}^{1st}/SF; \quad M_{all} = M_{max}^{1st}/SF. \qquad (4)$$

Substituting Eq.4 into Eq.3 it becomes possible to define the criteria in the form of the allowable sectional forces.

3. DESIGN AID FOR STIFFENED ARCH BRIDGE STRUCTURES

3.1 General Behavior

It is indicated herein that the design formulas for an arch are also applicable to stiffened arch bridge structures with some modification. In this case, the slenderness ratio λ_T of the whole structures is given by:

$$\lambda_T = L_s / \sqrt{(I_a + I_g)/A_a}$$

in which I_a = moment of inertia of an arch rib, I_g = moment of inertia of a stiffened girder, L_s = curvilinear length of an arch axis, and A_a = cross sec-

Table 1 Comparison of f-Results by the Design Formula with those by the Ultimate Strength Analysis (for application of the additional concentrated load).

r	$\bar{\lambda}$	q^{*}_{max}/q_{max}	M^{1st}_{max}/M_Y	N^{1st}_{max}/N_Y	f(ANALYSIS)/f(DESIGN)
0.5	1.24	0.719	0.813	0.333	1.098
	2.48	0.715	0.754	0.158	1.115
	3.73	0.736	0.640	0.090	1.056
0.75	1.24	0.801	0.586	0.479	1.086
	2.48	0.814	0.532	0.237	1.147
	3.73	0.829	0.610	0.130	1.247
0.99	1.24	1.000	0.130	0.746	1.124
	2.48	0.978	0.077	0.471	1.135
	3.73	0.968	0.047	0.243	1.139

$h/L = 0.15$, $\sigma_Y = 320$ N/mm^2, $Q_c = 50(1-r)q/3$.

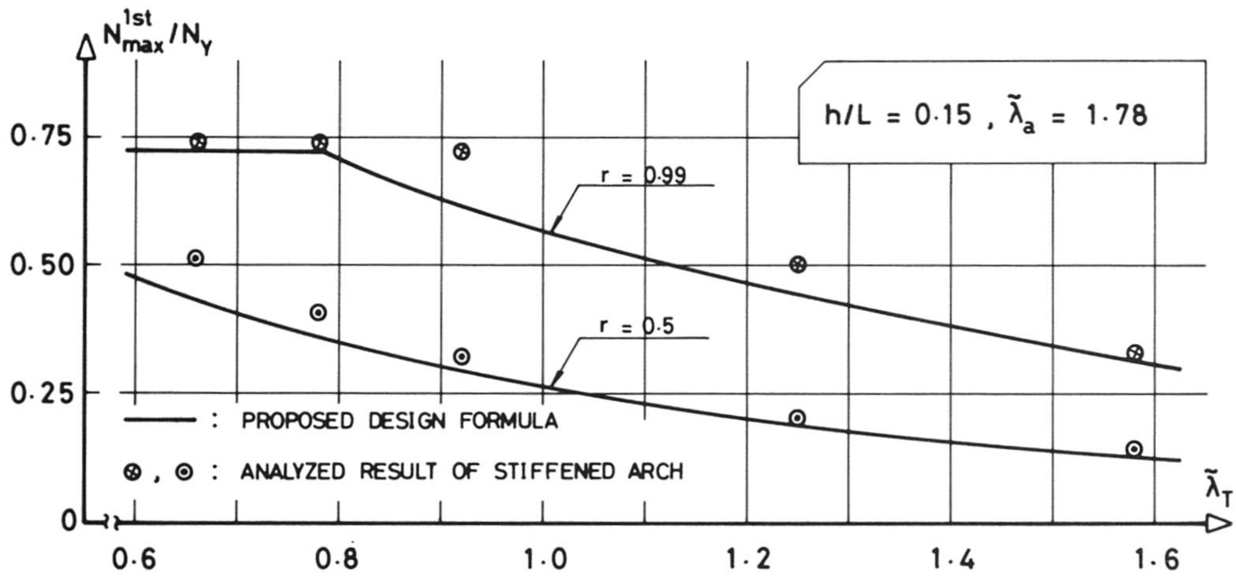


Fig.5 Comparison of Results by the Design Aid Equation for Stiffened Arch Structures with those by the Ultimate Strength Analysis.

tional area of an arch rib. Slenderness ratio of arch ribs λ_a and that of whole structures λ_T is varied in the following range:

$$\lambda_a = 200 \sim 600 \quad ; \quad \lambda_T = 95 \sim 260.$$

The stiffened arch structures adopted are bridge structures with two-hinged arch ribs and with stiffening girder of deck type. Loads on the arch ribs are applied through the posts. The following findings in Ref[4] may be useful in proposing a design aid for stiffened arch structures:

1) The bending moments acting on the arch rib and stiffening girder can be calculated approximately by dividing the total bending moments acting on the structure in proportion to their moments of inertia or their flexural rigidities;

2) The ultimate strength of the stiffened arch structures has analogous features to that of two-hinged arches if they do not behave localy - i.,e., local failure of the arch ribs, local buckling of the posts, etc -;

3) The ultimate strength of the stiffened arch structures is always higher than that of two-hinged arches if the slenderness ratio of a two-hinged arch equals the slenderness ratio λ_T of the stiffened arch structures. However, practically, the difference between the two may be not considerable within the above-mentioned investigated range;

4) It is generally not required to attention to the local failure of arch rib members under the unsymmetrical loading if a check is made for their local failures under uniformly distributed loads. The local failure strength of arch rib member can be determined by the so-called basic column strength curve - for example, the Japanese Specifications of Highway Bridges [1] and/or the SSRC[2] - when they have straight members. However, for curved members, it is advisable to estimate it by reducing 15% off.

3.2 Design Aid Expressions

By being reexamined the analytical results of the ultimate strength of the stiffened arches, and modified the Eqs.3, the ultimate strength of the stiffened arch structures may be evaluated by the following equations:

$$\lambda = \lambda_T \quad ; \quad \frac{N^{1st}_{max}}{N_Y} = \frac{N^{1st}_{a,max}}{N_{Ya}} \quad ; \quad \frac{M^{1st}_{max}}{M_Y} = \frac{M^{1st}_{a,max} + M^{1st}_{g,max}}{M_{Ya} + M_{Yg}} \quad \ldots\ldots\ldots\ldots \quad (5)$$

in which $N^{1st}_{a,max}$, $M^{1st}_{a,max}$ = maximum axial force and bending moment at a quarter point of an arch rib calculated by the 1st order elastic analysis for the stiffened arch structure under the ultimate load, $M^{1st}_{g,max}$ = maximum bending moment at a quarter point of a stiffening girder calculated by the 1st order elastic analysis for the stiffened arch structure under the ultimate load, N_{Ya} = squash axial force of an arch rib, and M_{Ya}, M_{Yg} = yield bending moment of an arch rib and a stiffening girder, respectively. Therefore, the maximum cross sectional forces of the stiffened arch structure are given by:

$$\frac{M^{1st}_{a,max}}{M_{Ya}} = \left(1 + \frac{M_{Yg}}{M_{Ya}}\right) \cdot \frac{I_a}{I_a + I_g} \cdot \frac{M^{1st}_{max}}{M_Y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6a)$$

$$\frac{M^{1st}_{g,max}}{M_{Yg}} = \left(1 + \frac{M_{Ya}}{M_{Yg}}\right) \cdot \frac{I_g}{I_a + I_g} \cdot \frac{M^{1st}_{max}}{M_Y} \quad ; \quad \frac{N^{1st}_{a,max}}{N_{Ya}} = \frac{N^{1st}_{max}}{N_Y} \quad \ldots\ldots \quad (6b),(6c)$$

In order to confirm the applicability of these design aid equations, the results and those by the ultimate analysis are compared by varying h/L, λ_a and λ_T. These slenderness ratio parameters are calculated, based on the buckling formula for two-hinged arches given by Stussi[2]:

$$\tilde{\lambda}_a = \lambda_a \sqrt{\frac{\sigma_Y \cos\gamma}{\alpha_k E R^2}} \qquad \lambda_T = \lambda_T \sqrt{\frac{\sigma_Y \cos\gamma}{\alpha_k E R^2}} \qquad (7)$$

in which α_k = buckling coefficient, γ = inclination angle of arch axis at the springing, and R = ratio of arch axis length to span length. The typical results are compared in Fig.5. Within the investigated range, the analyzed results for the ultimate strength is about 10% higher than those predicted from Eqs.6.

4. CONCLUSIONS

The maximum difference between the results given by the proposed design formulas and by the ultimate strength analysis is in 10% on conservative side and 5% on risky side with the practical structural range of steel arch bridges. Moreover, the design formulas yield conservative estimation for the case of 127/140 = 90% in all the results for the arch ribs and 94/100 = 94% for stiffened arch structures discussed herein. Since the design formula is found by so-called curve-fitting, it is possibly limitted in the adopted structural range. However, practical structural dimensions of steel arch bridges are sufficiently included in the domains discussed herein. Therefore, it could be concluded that the design formulas proposed by Eqs.3 and 6 evaluate the ultimate strength of two-hinged steel arch bridge structures accurately enougth for practical purpose and this design formulation may contribute a development in the ultimate strength design of steel arch bridge structures.

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