

**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte

**Band:** 51 (1986)

**Artikel:** Probabilistic appraisal of safety factors in design codes

**Autor:** Sugiyama, Toshiyuki / Fujino, Yozo / Ito, Manabu

**DOI:** <https://doi.org/10.5169/seals-39574>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

**Download PDF:** 19.03.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## Probabilistic Appraisal of Safety Factors in Design Codes

Appréciation probabiliste des facteurs de sécurité dans les normes de calcul

Wahrscheinlichkeitstheoretische Einschätzung von Sicherheitsfaktoren  
in Normen

### Toshiyuki SUGIYAMA

Associate Professor  
Yamanashi University  
Kofu, Japan

Toshiyuki Sugiyama, born in 1954, graduated and received his Dr. Eng. degree from the University of Tokyo. He is now engaged in teaching and research on structural engineering, particularly the application of reliability theory at Yamanashi University.

### Yozo FUJINO

Associate Professor  
University of Tokyo  
Tokyo, Japan

Yozo Fujino, born in 1949, graduated from the University of Tokyo, and received his Ph.D. degree at the University of Waterloo, Canada in 1977. After teaching at the University of Tsukuba, he is now at the University of Tokyo.

### Manabu ITO

Professor  
University of Tokyo  
Tokyo, Japan

Manabu Ito, graduated in 1953, and received his Dr. Eng. degree from the University of Tokyo. He has been engaged in teaching and research on bridge engineering and structural dynamics at the same university.

### SUMMARY

This contribution discusses the reliability level for the determination of safety factors in structural codes. The first part is concerned with the target safety level of a component of a structural system, taking into account the variability of loads and uncertainties due to gross errors. The latter part of the paper discusses the balance of safety levels of different components in a structural system using a long-span suspension bridge as an example.

### RÉSUMÉ

L'article traite du niveau de fiabilité pour la détermination de facteurs de sécurité dans les normes de construction. La première partie concerne le niveau de sécurité optimal pour un élément du système structural, considérant la variation des charges et l'incertitude possible due à de grossières erreurs. La deuxième partie envisage l'équilibre des niveaux de sécurité de divers éléments dans un système structural, à partir de l'exemple d'un pont suspendu de grande portée.

### ZUSAMMENFASSUNG

Der Beitrag befasst sich mit dem Zuverlässigkeits-Niveau für die Festlegung von Sicherheitsfaktoren in Tragwerks-Normen. Zunächst befasst er sich mit dem anzustrebenden Sicherheits-Niveau von Bauteilen, wobei sowohl die Streuungen der Lasten als auch die Unsicherheiten aus groben Fehlern berücksichtigt werden. Sodann wird am Beispiel einer weitgespannten Hängebrücke die Ausgewogenheit des Sicherheits-Niveaus verschiedener Bauteile eines Tragsystems erörtert.



## 1. INTRODUCTION

One of the most important items in reliability-based code making is to determine the target safety level. It is often believed in code calibration that the uniform safety level for different loads or their combination is desirable[1]. However, it seems that the situation is not necessarily so in the current design codes. For example, the safety factor or safety level for dead plus live loads is, in most of the codes, taken higher than that for the environmental loads such as wind or earthquake effects[2].

The target safety level is basically determined such that the total cost including the initial and failure costs is minimized. It should be noted that the variability of the loads would considerably affect the initial cost in securing the required safety level, and hence the optimal safety level can be expected to depend on the load variability.

The history of structures indicates that incompleteness of engineer's knowledge or human error has been one of the major causes of the structural failures[3]. It would be true that the traditional safety factor has been expected to cover not only randomness of resistance and load, but also partly to cover the above mentioned uncertainties which are called as 'gross errors' in this paper. Certain gross errors cannot be completely excluded in the present design and construction processes, although their occurrence may be kept below a prescribed level by means of quality control and inspection.

Under these considerations, in the first part of this paper the target safety level of a component of structural system is assessed, taking into account the variability of loads and uncertainties due to gross errors.

Many of civil engineering structures are regarded as a system of several components. The safety levels or safety factors should not be necessarily the same for different load combinations or different structural components[4]; the appropriate safety factors for each component should be chosen depending upon its cost and consequence of its failure. In the second part of this paper, the balanced allocation of safety factors in a structural system is discussed on the basis of the economic optimization. A long-span suspension bridge is used therein as an example.

## 2. TARGET SAFETY LEVEL

### 2.1. Target safety level by cost minimization principle

#### 2.1.1 Evaluation of total cost

Total cost  $C_T$  of civil engineering structure may be expressed by

$$C_T = C_I + P_F C_F \quad (1)$$

where  $C_I$  and  $C_F$  represent the construction plus maintenance cost and failure cost of the structure (component), respectively, while  $P_F$  is the probability of failure[5]. According to the principle of total cost minimization, the design of structure attaining the minimum total cost is regarded as optimum.

It is assumed in this study that both the structural resistance  $R$  and the load effect  $S$ , treated as random quantities, are log-normally distributed. Then the probability of structural failure  $P_F$  is given by

$$P_F = \Phi(-\beta) \quad (2)$$

where  $\Phi(\cdot)$  is the cumulative distribution function of the standard normal distribution and safety index  $\beta$  is

$$\beta = \{ \ln(\bar{v}\sqrt{1+V_S^2}/\sqrt{1+V_R^2}) \} / \sqrt{\ln\{(1+V_R^2)(1+V_S^2)\}} \tag{3}$$

$$\bar{v} = \bar{R} / \bar{S} : \text{central safety factor} \tag{4}$$

$\bar{R}$ : mean of R,  $V_R$ : coefficient of variation of R,  
 $\bar{S}$ : mean of S,  $V_S$ : coefficient of variation of S

In general, initial construction cost  $C_I$  increases with the increase of the safety factor  $v$ . The following function proposed in Refs. [6] and [7] is used:

$$C_I(v) = C_I(v_0) \{ 1 + b(v/v_0 - 1) \} \tag{5}$$

where  $v_0$  is the value adopted in the current code and 'b' is a constant. Then the total cost is obtained as

$$C_T = C_I(v_0) \{ 1 + b(\bar{v}/\bar{v}_0 - 1) + P_F C_F^* \} \tag{6}$$

Failure cost of structure should be evaluated taking account of the direct loss and the indirect loss associated with social and economical effects caused by failure. Because its evaluation is very difficult at present, the dimensionless failure cost  $C_F^*$  is assumed to be constant in this study.

Design resistance  $R_d$  and design load  $S_d$  are assumed to be the fractile values, corresponding to 10% lower and upper fractile, respectively. Furthermore, the following design format is used:

$$R_d / v \geq S_d \tag{7}$$

Then substituting eq. (2) into eq. (6) and satisfying the condition  $dC_T/d\beta = 0$ , the optimal safety level  $\beta_{opt}$  can be obtained.

2.1.2 Numerical results and discussions

Fig. 1 shows the calculated optimal safety level  $\beta_{opt}$  as a function of the coefficient of variation,  $V_S$  of load effect. It is found that the value of  $\beta_{opt}$  for the small  $V_S$  is significantly larger than  $\beta_{opt}$  for the large  $V_S$ . This result suggests to assign a relatively high target safety level to the structures subject to the less variable load effects. The reason for being obtained this result is that the safety index  $\beta$  does not effectively increase with the increase of the central safety factor when the value of  $V_S$  is large. In other words, the large initial cost is required to achieve high safety level for this case.

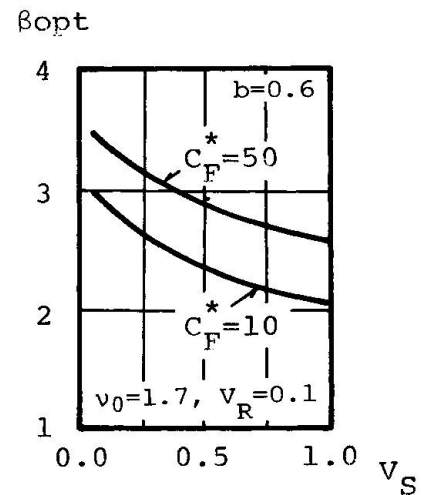


Fig.1 Optimal safety level  $\beta_{opt}$

2.2 Target safety level in presence of gross errors

2.2.1 Probabilistic model

It is well recognized that structural safety depends not only on statistical uncertainties but also uncertainties due to gross errors such as human errors. Then the effect of the latter uncertainties on structural reliability is investigated by use of the simple probabilistic model defined as below[3].

Assume that the structural resistance  $R$  decreases to  $R_u$  because of the existence



of uncertainties due to gross errors and that  $R_U$  is defined as

$$R_U = R \times U \quad (8)$$

where  $U$  is the random variable whose probability density function is

$$f_U(x) = P\delta(x-\phi) + (1-p)\delta(x-1) \quad (9)$$

in which  $\delta(\cdot)$  is Dirac's delta

function. In this modeling it is considered that the resistance reduction occurs with probability  $p$  and that structural resistance  $R$  deteriorates to  $(1-\phi)R$  as shown in Fig. 2. The probability of failure with log-normally distributed  $R$  and  $S$  is expressed as

$$P_F^* = \text{Prob}[R_U < S] = pP_{F_U} + (1-p)P_{F_N} \quad (10)$$

in which

$$P_{F_U} = \Phi(-\beta_n - \ln\phi / \sqrt{\ln\{(1+V_R^2)(1+V_S^2)\}}) \quad (11)$$

$$P_{F_N} = \Phi(-\beta_n) \quad (12)$$

$\beta_n$  in eqs. (11) and (12) is the 'apparent' or 'operational' safety index and is the same as that defined by eq.(3). If the values of  $p$  and  $\phi$  in eqs. (10) and (11) are given,  $P_F^*$  can be calculated by eq. (10).

### 2.2.2 Values of parameter $p$ and $\phi$

We attempt herein to estimate the reasonable range of parameters  $p$  and  $\phi$  from the surveys on bridge failures[8]. Suppose that all of structures are designed so as to attain the target safety level  $\beta_n$ . Under this condition, the ratio 'a' which leads to the relation between the number of failure caused by uncertainties due to gross errors and that caused by statistical ones is defined from eq. (10) as

$$a = pP_{F_U} / (1-p)P_{F_N} \quad (13)$$

According to the structural failure data in Ref. [8], it can be found that the ratio 'a' takes the values between 0.25 and 2.4. On the other hand, many civil engineers and investigators consider that the actual failure probability of civil engineering structures may be higher than operational one[9], which is generally said to be about  $10^{-4} \sim 10^{-6}$ . Taking account of these situations, the ratio 'a' is assumed here to take the value between 0.25 and 10.0. Consequently, the fluctuation range of parameter  $p$  is  $4.1 \times 10^{-3} \sim 1.4 \times 10^{-1}$  for  $\phi = 0.7$  under the conditions that  $V_R = 0.1$ ,  $V_S = 0.2$  and  $\beta_n = 3.0$ .

### 2.2.3 Numerical results and discussions

The apparent  $\beta_n$  to attain the probability of failure  $P_F^* = 1.35 \times 10^{-3}$  ( $\beta_n^* = \phi^{-1} (1.35 \times 10^{-3}) = 3.0$ ) is calculated under the conditions that  $\phi = 0.7$  and  $P = 4.1 \times 10^{-3} \sim 1.4 \times 10^{-1}$  and are presented in Fig. 3. Fig. 3 shows that the case dominated by less variable load effects requires higher target  $\beta_n$  to attain the same safety level. This means that the probability of failure of the structure subject to less variable loads is strongly increased by the existence of the gross errors. In other words, uniform apparent safety level results in the less safety margin against the uncertainties due to gross errors in the case of the small variance of load effects.

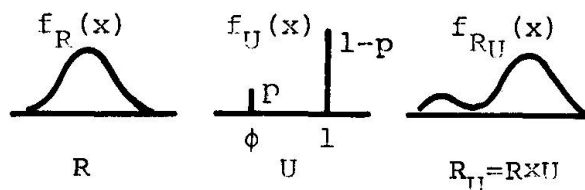


Fig.2 Structural resistance deterioration model

2.3 Target safety level taking account of both total cost minimization and uncertainties due to gross errors.

In this section, target safety level taking account of both total cost minimization and uncertainties due to gross errors is discussed. Substituting eq. (10) instead of eq. (2) into eq. (6) and applying the principle of total cost minimization,  $\beta_n$ , opt is calculated. The parameter values used are;

$$C_F^* = 10 \text{ and } 50, v_0 = 1.7, V_R = 0.1, V_S = 0.05 \sim 1.0,$$

$$\phi = 0.7, p = 4.1 \times 10^{-3} \sim 1.4 \times 10^{-1}, b = 0.6$$

Fig. 4 shows the optimum apparent safety level  $\beta_n$ , opt calculated as a function of the coefficient of variation of load effect. As expected,  $\beta_n$ , opt is larger than that presented in Fig. 1. And it is also found that  $\beta_n$ , opt is considerably higher for smaller coefficient of variation  $V_S$  of the load effect. This result agrees with that obtained from Fig. 1 and Fig. 3. Accordingly, it can be concluded that relatively higher safety level to the less variable load effects should be assigned and consequently that it is not necessary to provide large difference between load factors according to the variability of load effects.

3. BALANCE OF SAFETY FACTORS IN A STRUCTURAL SYSTEM

Many of the civil engineering structures are consisted of different components and have various failure modes. Hence they are to be treated as structural systems. Long-span suspension bridge is certainly one of the typical structural systems. Its superstructure is composed of towers, cables and stiffening girder. In towers and cables of a long-span suspension bridge, dead load effect exceeds 90% of the total design load effect. On the other hand, the design of the principal members of stiffening girder is controlled by the wind load, at least in our Japanese practice. This is especially true for the truss type of girders because of the large wind force. The dead load has a very small variation under elaborate quality control, but the wind load and the performance of the suspension bridge under wind action is very much uncertain. This observation leads to smaller safety factor for the towers and cables and to larger safety factor for the stiffening girder. The current design specification for long-span suspension bridges seems opposite; approximately the safety factor of 3.0 for the ultimate strength of the cable and less than 2.0 for the stiffening girder[10].

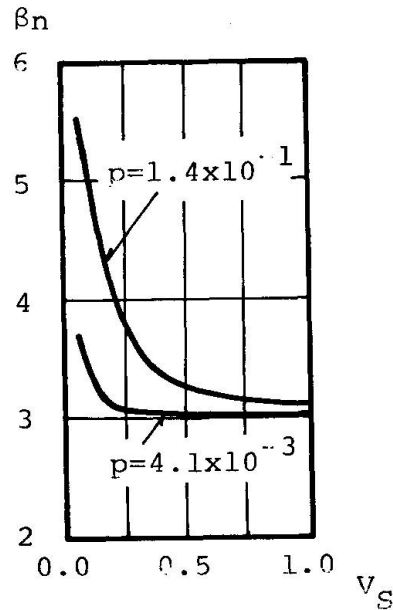


Fig.3 Apparent safety level  $\beta_n$

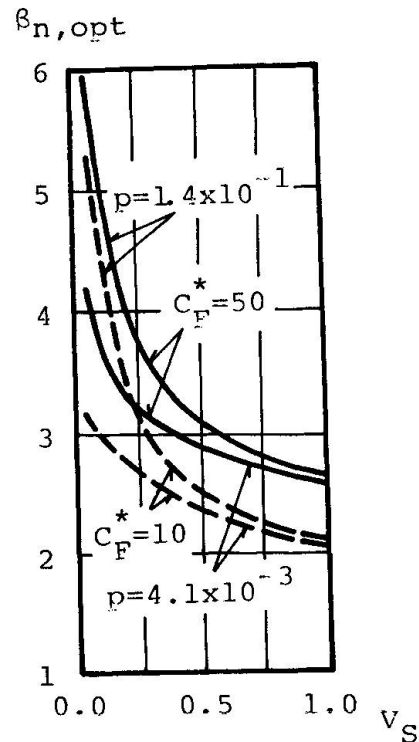


Fig.4 Optimal apparent safety level  $\beta_n$ , opt



In this section, balanced allocation of the safety factors in a system is studied again from an economical point of view. Only two components in the suspension bridge, namely cables and stiffening girder are investigated herein.

**3.1 Evaluation of total cost**

In a similar manner as in section 2, the total cost of each component is expressed as

$$C_{TC} = C_{IC} + P_{FC} C_{FC}, \quad C_{TG} = C_{IG} + P_{FG} C_{FG} \tag{14}$$

in which subscript 'C' and 'G' stand for the cables and girder, respectively.

Then the total cost  $C_T$  of the structure is given by

$$C_T = (C_{IC} + P_{FC} C_{FC}) + (C_{IG} + P_{FG} C_{FG}) \tag{15}$$

The random variables, i.e., structural resistance, dead and wind loads are assumed log-normally distributed and then the probability of failure  $P_{FC}$  and  $P_{FG}$  can be obtained from eq. (2).  $C_{IC}$  and  $C_{IG}$  are assumed to be the form of eq. (5). Break of the cable leads to the collapse of the suspension bridge; failure of the stiffening girder does not necessarily mean the collapse of the whole structure. Considering this, it is reasonable to assume  $C_{FC}$  is larger than  $C_{FG}$ . The sum of  $C_{IC}$  and  $C_{IG}$  is assumed to keep constant.

**3.2 Numerical results and discussions**

Numerical computations are carried out in order to find the optimal set of safety factors  $v_C$  and  $v_G$ .

Under these observations as well as the design calculation used in the proposed Akashi Straits Bridge, the longest bridge of the Honshu-Shikoku project, the following values are subjectively chosen and used in the example calculation:

$$v_0 = 1.7, \quad C_{IC} : C_{IG} = 2 : 1, \quad C_{FC} : C_{FG} = 2 : 1, \quad C_{IC} : C_{FC} = 1 : 100, \quad v_{RC} = v_{RG} = 0.1,$$

$$v_D = 0.1 \text{ (dead load effect)}, \quad v_W = 0.3 \text{ and } 0.5 \text{ (wind load effect)}$$

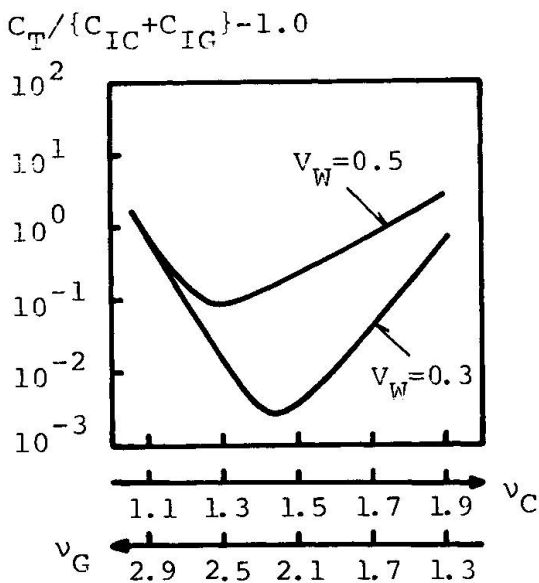


Fig.5 Total cost vs. Safety factors  $v_C$  and  $v_G$

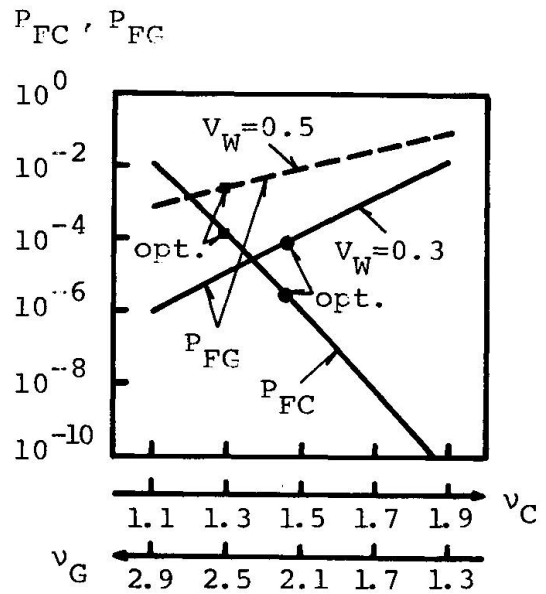


Fig.6  $P_{FC}$  vs.  $v_C$  and  $P_{FG}$  vs.  $v_G$



The dimensionless total cost  $C_T(v_C, v_G) / \{C_{IC}(v_0) + C_{IG}(v_0)\} - 1.0$  and probability of failure  $P_{FC}$  and  $P_{FG}$  were calculated as the function of  $v_C$  and  $v_G$  as shown in Fig. 5 and Fig. 6, respectively. It can be found that the optimal safety factor  $v_{C,opt}$  is noticeably smaller than  $v_{G,opt}$ , while the optimal value of  $P_{FC}$  corresponding to  $v_{C,opt}$  is smaller than optimal  $P_{FG}$  corresponding to  $v_{G,opt}$ . Although  $P_{FC}$  decreases remarkably as the increase of  $v_C$ , further increase of  $v_C$  is not profitable referring to Fig. 5. The combination of  $v_C$  and  $v_G$  in the current design specifications for long-span suspension bridges contradicts with the above findings. Although the parameter values need to be more carefully chosen, reconsideration of the selection of safety factors might be desirable.

#### 4. CONCLUDING REMARKS

The target safety level of a structural component as well as structural system has been discussed. Introducing a gross error model, the optimal safety level of a structural component was calculated on the basis of cost optimization.

The results show that it is reasonable to assign relatively higher safety level to the less variable load effects. This implicitly supports the situation of safety level in the current design code.

Employing a long-span suspension bridge as a structural system example, the optimal allocation of the safety levels for different components was studied. The numerically higher safety level for the components subject to less variable load effects is found optimal in this example as well; namely, the higher safety level for the cable and the lower for the stiffening girder. The reconsideration of the safety factors in the current design practice of long-span suspension bridges is suggested on the basis of these findings.

#### REFERENCES

- [1] RAVINDRA, M.K. and R.V. GALAMBOS: Load and Resistance Factor Design for Steel, Proc. of ASCE, Jour. of the Structural Division, Vol. 104, No. ST9, pp. 1337-1353, Sept., 1978.
- [2] ELLINGWOOD, E., T.V. GALAMBOS, J.G. MACGREGOR and C.A. CORNELL: Development of a Probability-Based Load Criterion for American National Standard A58-Building Code Requirements for Minimum Design Loads in Buildings and Other Structures, U.S. Dept. of Commerce, SP-577, pp. 1-222, June, 1980.
- [3] NOWAK, A.S.: Effect of Human Error on Structural Safety, ACI Journal, pp. 959-972., 1979.
- [4] LIND, N.C., L.I. KNAB, and W.B. HALL: Economic Study of the Connection Safety Factor; Proc. of Third Specialty Conference on Cold-formed Steel Members, pp. 951-966.
- [5] TURKSTRA, C.J.: Theory of Structural Design Decisions, Solid Mech. Division, Univ. of Waterloo, SM Study, No. 2, pp. 1-124, 1970.
- [6] LIN, S.: Reliability-based Design Code for Highway Bridges, Master Thesis, Univ. of Tokyo, pp. 1-100, March, 1980 (in Japanese).
- [7] LIND, N.C.: Approximate Analysis and Economics of Structures, Proc. of ASCE, Jour. of the Structural Division, Vol. 102, No. ST6, pp. 1177-1196, June, 1977.
- [8] SMITH, D.W.: Bridge Failures, Proc. of ICE, Vol. 60, Part 1, pp. 367-382, Aug., 1976.





- [9] BROWN, C.B.: A Fuzzy Safety Measure, Proc. of ASCE, Jour. of the Eng. Mech. Division, Vol. 105, No. EM3, pp. 855-871, June, 1979.
- [10] HONSHU-SHIKOKU BRIDGE AUTHORITY; Design Specifications for Superstructure, August, 1977.