Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte
Band: 51 (1986)

Artikel: Probability-based working stress design code
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DOI: https://doi.org/10.5169/seals-39575

Probability-Based Working Stress Design Code

Dimensionnement probabiliste sur la base de contraintes admissibles

Probabilistisch hergeleitetes System von zulässigen Spannungen

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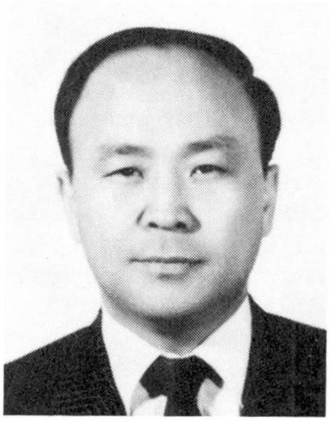

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SUMMARY

Reliability of reinforced concrete structural members is evaluated by using an advanced second moment reliability method. Then, a practical method for code calibration is shown in this paper. A set of allowable stresses for reinforced concrete is proposed based on the rational target reliability indices.

RÉSUMÉ

La fiabilité des éléments de structure en béton armé est calculée sur la base de la méthode de fiabilité du deuxième ordre. Une méthode pratique de calibrage de la norme est présentée. Des contraintes admissibles pour les constructions en béton armé sont recommandées sur la base d'indices de fiabilité déterminés.

ZUSAMMENFASSUNG

Die Zuverlässigkeit von Stahlbeton-Tragelementen wird aufgrund einer verfeinerten Methode der Zuverlässigkeitstheorie ermittelt. Eine praktische Methode zur Kalibrierung von Normen wird vorgestellt. Schliesslich wird ein Satz von zuverlässigen Spannungen für die Bemessung von Stahlbeton angegeben, der sich auf das angestrebte Zuverlässigkeits-Niveau stützt.

1. INTRODUCTION

Recent advances in structural code developmemt are primarily directed toward probability-based framework of limit state design (LSD) or load and resistance factor design (LRFD). However, little attention has been paid to the systematic development of a probability-based working stress design (PBWSD) code, despite the fact that WSD is still predominantly used in design practice in those countries such as Korea and Japan among others.

PBWSD code, like LSD or LRFD codes[5,9,13], could employ up-to-date advanced first order second moment (AFOSM) reliability methods, and thus can be drafted as an equivalent probability-based code which provides essentially identical and consistent reliability. On the one hand, it is generally recognized that WSD has some serious drawbacks compared to LSD or LRFD-multiple factor design methods. It appears that the most serious drawback of WSD is the missing of the flexibility that the presence of many adjustable load factor gives, and the feeling about the overloading safety of each variable load. And thus, in the long run, LSD or even more advanced or higher level reliablity based design codes in the future should be considered as the prototype structural code for the next generation. On the other hand, it is expected that it will take more than a decade for most practitioners to abandon WSD and become familiar with LSD or LRFD in design practice especially in those countries such as Korea or Japan. Therefore, for the transition decade to come the current WSD code should be remodeled as a pobability based equivalent design method corresponding to the LRFD or LSD code.

This paper presents a practical procedure for the calibration of the PBWSD code for reinforced concrete, reports the investigations of the structural reliability by the current code and then proposes pobability-based safety provisions for the WSD code.

2. CURRENT CODE AND DESIGN PRACTICE

R.C. design standards in Korea are almost similar to the ACI code, and specify two alternative design methods, that is, the strength design method and WSD which are not probability-based. Although the current code recommends the use of the strength design method in design practice, the majority of engineers still prefers to use WSD and sticks to the concept of safety in terms of allowable stresses or traditional notional safety factors. Virtually no engineers in practice have a little understanding of modern structural reliability or safety concepts. From a probabilistic point of view based on a series of investigations, the safety provisions of the current R.C. design standards are irrational and invariably too much conservative, and, in general, result in uneconomical designs although some safety provisions are too low or fluctuating too much. For inatance, in case of the usual sustained service design loads (dead + sustained live load), the allowable stresses are given as a fraction of nominal strength of materials, whereas, for safety checking with the load combination with transient loads such as wind or earthguake loads, the exceedence of 1/3 of the allowable stresses is provided in the code primarily based on the experiences and judgements as in usual traditional WSD codes.

3. RELIABILITY BASIS FOR WSD CODE

3.1 Simplified procedure for Parameters

Almost every prlbability-based code model employs advanced or practical Level II AFOSM reliability methods for the derermination of safety parameters in the code calibration procedure. AFOSM methods are well known and established reliability methods, and the detailed concepts and the procedure of which are published in the major reports [9,16,18], papers [6,7,10,11] and texts [12,20]. No detailed or even brief procedures or equations will be presented in this paper.

Once a LRFD format code is drafted by employing an AFOSM Level II methods [9,15], then nominal safety factors, n', for each limit state can be obtained directly from the safety factor parameters (ϕ, γ_{si}) of the LRFD code. In this paper, a simplified procedure for determining ϕ, γ_{si} is also briefly presented as reference. Suppose we choose the following format as a LRFD limit state equation [8,9];

$$\phi\bar{R} = \Sigma \gamma_{si} \bar{S}_i \tag{1}$$

where, $\bar{R}, \bar{S}_i$ = mean resistance and ith load effects

$$\phi = \exp(-\alpha_R \beta_o V_R) \tag{2a}$$

$$\gamma_{si} = 1 + \alpha_{si} \beta_o V_{si} \tag{2b}$$

in which V_R, V_{si} ; coefficients of variation of R & S_i

α_i ; direction cosine of design point on failure surface

β_o ; target reliability index

Also note that, in terms of the total load factor design format, the safety parameters are :

$$\phi\bar{R} = \gamma_s \bar{S} \tag{3}$$

where,
$$\gamma_s = 1 + \alpha_S \beta_O V_S = 1 + \frac{V_S^2 \beta_O}{\sqrt{\gamma_s^2 V_R^2 + V_S^2}} \tag{4}$$

Note that, by definition, the central safety factor $n_o = \bar{R}/\bar{S} = \gamma_S/\phi$. If we make use of the relationship between γ_s and γ_{si}, which can be derived directly [3] as:

$$\gamma_{si} = 1 + \frac{\rho_i V_{si} \beta_o}{(1+\Sigma\rho_i)\sqrt{\gamma_S^2 V_R^2 + V_S^2}} \tag{5}$$

where, $\rho_i = \bar{L}_i/\bar{D}$, in which $\bar{L}_i$=ith variable load, $\bar{D}$=dead or permanent load, then the safety parameters ϕ, γ_{si} of a LRFD code can be determined from Eq.(3)-(5) provided that the total load factor γ_s is evaluated iteratively by using Eq.(4).

Once ϕ, γ_{si} corresponding to a target reliability index β_o are evaluated, then the nominal safety factor n' can be obtained in terms of the mean-nominal ratio of resistance and load effects ($\eta_R = \bar{R}/R'$, $\eta_S = \bar{S}/S'$) as follows.

$$n' = \frac{\eta_S}{\eta_R} n_o \tag{6}$$

The nominal values of R' and S' may be obtained from the characteristic values of basic random variables.

3.2 Allowable Stresses

Allowable stresses in WSD are usually expressed as a fraction of material strengths by using "notional safety factor", n, newly defined in this paper. It is evident that the notional safety factor of WSD is, in general, different from the nominal safety factor, n', of limit state codes defined according to failure modes [3] as in the previous section (for instance, $n = (M_y/M_n) n'$).

3.2.1 Flexural Member

-1. Bending : Although the allowable stress of steel of a R.C. beam can be simply expressed as $f_{sa} = f_y/n'$, the allowable stress of concrete can not be given as f_c'/n'. And thus, in this paper, a simple but rational way of determining the allowable stress of concrete which results in under-reinforced section (in the limit state sense) but a balanced section (in the WSD sense) is presented. Suppose we prefer to proportion a R.C. beam so that the reinforcing ratio of the section takes near 1/2 p_{max} or optimum ratio p_o as an under-reinferced section. Then, the allowable stress of such a section can be derived by using balanced section formula [3], that is,

$$p_b = \frac{f_{ca}}{2f_{sa}}\left(\frac{f_{ca}E_s/E_c}{f_{sa}+f_{ca}E_s/E_c}\right)$$

as

$$f_{ca} = p_b f_{sa} + \sqrt{p_b^2 f_{sa}^2 + 2p_b f_{sa}^2 E_s/E_c} \qquad (7)$$

where, $p_b = \frac{1}{2} p_{max}$ or p_o (optimun steel ratio)

$f_{sa}=0.85f_c'm/n'$, in which m is the effective strength ratio.

-2. Shear: It is clear that the allowable shear stress of concrete can be directly obtained from $\tau_a = \tau_{c1}/n'$ or $\tau_a = \tau_{c2}/n'$ respectively for one way or two way action, and the allowable stress of shear reinforcement as $f_{sa}=f_y/n'$.

3.2.2 Compression Member

In case of pure compression, the allowable stress of concrete and steel are simply given as $f_{ca} = 0.85f_c'/n'$, $f_{sa} = f_y/n'$, respectively, but, due to the complexity of reliability analysis of general columns subject to compression with bending, the outline of the reliability procedure for those columns can not be presented herein, although this study made use of the previous study [4, 8] which is not rigorous but approximate and practical. The essential part of the column design provisions for the PBWSD is to construct the allowable linear interaction diagram based on the limit state interaction diagram by using the nominal safety factor as proportional reduction factor. However, the more rational way of provisioning R.C. columns in the PBWSD is to adopt the limit state column design procedure by simply taking the permissible resistances as $P_a = P_n/n'$, $M_a = M_n/n'$, and thus using the column interaction equation or diagram along with the service load effects (P,M).

3.2.3 Retaining Wall

The limit states of the stability for retaining walls are overturning, sliding and bearing capacity, which can be formulated in terms of dead weights, soil pressures and surcharge loads. Once the parameters for each limit state corresponding to the selected set of target reliability indices are obtained, then the nominal safety factor, n', for each stability limit state of retaining walls can be obtained from the corresponding parameters [3]. Also, note that the allowable stresses of R.C. retaining walls can be obtained by following the same way as in the case of flexural members but with different load effects and target reliability indices.

4. CALIBRATION OF PBWSD CODE

4.1 Statistical Uncertainties

Statistical uncertainties of resistances as shown in Table 1 are evaluated from the best available data in Korea [1,2,3,4]. However, uncertainties of load varibles are chosen as conservative values mainly based on the engineering judgements and experiences as well as the available foreign data [8,9,14], because the statistical load data at present are not available and the research on stochastic load models is still going on in Korea.

4.2 Reliabilities of R.C. Members Designed by the Current Code

Figure 1 shows the reliability index of the various R.C. structural members designed by the current WSD code, As shown in the figure, the reliability of

Table 1 Resistance & Load Statistical

Action	Type	V_R	V_{SD}	V_{SL}	V_{SL}	V_W	$\bar{R}/R_n$	$\bar{D}/D_n$	$\bar{L}/L_n$	$\bar{L}/L_{In}$	$\bar{W}/W_n$
Bending	Beams	0.16	0.10	0.26	0.50	0.37	1.12	1.05	1.20	0.50	0.90
	2-way Slabs	0.16	0.24	0.35	0.50	0.37	1.12	1.05	1.20	0.50	0.90
	Footings	0.18	0.10	0.26	0.50	0.37	1.12	1.05	1.20	0.50	0.90
Shear	Beams (Flex.)	0.17	0.10	0.26	0.50	0.37	1.09	1.05	1.20	0.50	0.90
	2-way Slabs	0.17	0.24	0.35	0.50	0.37	1.09	1.05	1.20	0.50	0.90
	Footings	0.19	0.10	0.26	0.50	0.37	1.09	1.05	1.20	0.50	0.90
Compress-ion	Tied Col.	0.17	0.10	0.26	0.44	0.37	1.05	1.05	1.20	0.50	0.90
	Spiral Col.	0.17	0.10	0.26	0.44	0.37	1.05	1.05	1.20	0.50	0.90
Stability (Retain-ing Wall)	Overturning	0.09	-	0.26	V_{SS}=0.20		1.19	0.99	1.34	S/S_n=1.14	
	Sliding	0.14	-	0.25	V_{SS}=0.16		1.18	0.99	1.34	S/S_n=1.14	
	Bearing Cap.	0.44	0.06	0.21	V_{SS}=0.08		1.17	0.99	1.34	S/S_n=1.14	

various R.C. members is, in general, invariably conservative, and fluctuate to a considerable degree depending on mean live to dead load ratio ($\rho=\bar{L}/\bar{D}$). It can be easily observed that a design by the current WSD code results in the irrational and uneconomical proportioning, and the reliability is fairly sensitive to the variation of the load ratio, which is the inevitable pitfall of WSD with single safety parameters.

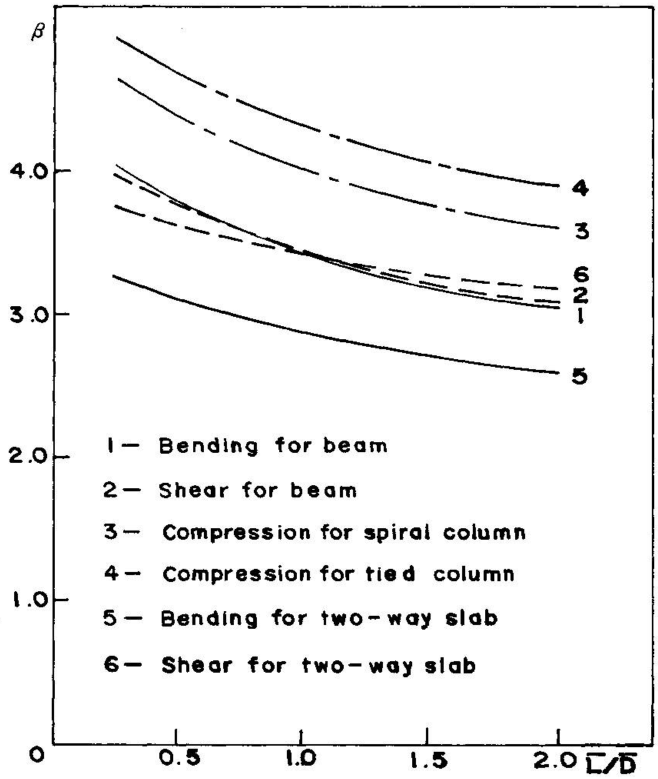

Fig. 1 Reliability Indices of R.C. Members Designed by the Current WSD Code (β v.s. $\bar{L}/\bar{D}$)

4.3 Selection of Target Reliability Indices

No established procedure for the rational selection of target reliability indices, however, is available so far, although various approaches have been suggested in the several procedure reports such as CIRIA report 63 [16] and NBS SP-577 [9] , and a few papers [17,19] among others. The socio-economic criteria approach adopted by the CIRIA report still needs further investigation, but the method of calibration against the current practice used by the NBS report may not also provide optimal target reliability indices due to the lack of rationale behind the selection criteria. A research on the selection of optimum target reliability based on sensitibity analysis and optimization method is still on the way. In the mean time the approach proposed in this study is, therefore, based on the concept of the desired hierarchy of safety level along with the engineering judgement and experiences as well as foreign practices together with the trade-off between theory and practice, and may be briefly stated as follows:

- Set up the desired hierarchy of safety levels for each limit state of each structural component (e.g. slab < beam < column < footing , flexure < shear , tied < spiral).

- Consider the reliability level of the current practice, and review the rationality of the current reliability based on engineering judgements.
- Based on the deviation of β_0 between limit states used in the foreign codes, and by the judgement, select the tentative β_0 for each limit state.
- Curry out the calibrations and examine the results whether it is reasonable or acceptable after a few cycle of adjustment, and then select the final set of desired target reliability indices.

Table 2 shows the results of selected reliability indices obtained by the above arguments

Table 2 Allowable Stresses and Nominal Safety Factors

Action	Type	Target Reliability Indices	Allowable Stresses*				n'	n	γ_0
			Concrete		Steel				
			Cur.	Pro.	Cur.	Pro.			
Bending	Beams	3.0(2.5)	0.40	0.45	0.50	0.55	1.84	1.79	0.70
	2-way Slabs	2.8(2.5)	0.40	0.45	0.50	0.55	1.95	1.89	0.63
	Footing	4.0(3.5)	0.40	0.40	0.50	0.45	2.39	2.32	0.69
Shear	Beams(Flex.)	3.2(2.7)	0.47	0.50	0.50	0.50	2.02	2.02	0.69
	2-way Slabs	3.0(2.7)	0.42	0.45	0.50	0.45	2.14	2.14	0.63
	Footings	4.2(3.7)	0.42	0.40	0.50	0.40	2.65	2.65	0.69
Compression	Spiral Col.	3.5(3.0)	0.25	0.35	0.40	0.40	2.46	2.46	0.70
	Tied Col.	4.0(3.5)	x85%	x90%	x85%	x90%	2.65	2.65	0.69
Stability (Retaining Wall)	Overturning	4.0	-	-	-	-	1.80	-	-
	Sliding	3.5	-	-	-	-	1.90	-	-
	Bearing Cap.	3.0	-	-	-	-	3.60	-	-

* allowable stresses = (factors in concrete and steel)x(nominal strength)

4.4 Proposed Safety Provisions for WSD Code

Table 2 shows essential parts of the summary of the calibration results of the safety parameters for the following PBWSD format:

$$R_a'(f_{ca}, f_{sa}) \geq \gamma_o \sum S_i' \tag{8}$$

,where γ_0 ; load combination factor for the combinations other than D+L, which is the ratio of n' for $(D+L_I+W)$ and n' for (D+L).

At first, the nominal safaty factors,n', the corresponding notional safety factors, n,and the load combination factor, γ_0 , are calculated, as shown in Table 2, by following the procedure of Eq.(1)-(6) with the selected target reliability indices shown in Table 2 and the uncertainties shown Table 1. It can, thus, be seen that γ_0 shown in the last column result in near 0.7 except γ_0 of slabs (=0.63). It can, then, be concluded that a bit conservative value γ_0 =0.7 could be satisfactory as the load combination factor in practice $(0.7x(D+L_I+W))$. Next, it can, also, be admitted that the factors for the calculated allowable stresses are to be rounded up as the proposed nominal values as shown in Table 2 for the convenience of the use in practice. Note that in the calibration of the nominal safety factors and allowable stresses, the weighted error minimization which is widely accepted in the code calibration is used in this study, and an optimum degree of complexity in the matrix of safety factors is considered as shown in Table 2.

4.5 Comparision with the Other Codes

First, following observations can be made by comparing the proposed PBWSD provisions with the current WSD provisions as shown in Table 2. The allowable stresses of the current code provisions which are obtained, mainly from the engineering judgements and experiences are significantly different in a number of cases from those of the proposed PBWSD provisions as shown in Table 2. For instance, the proposed allowable concrete stress of column is $0.35f_c'$, while the current is $0.25f_c'$, which indicates 40% difference, and in the case of stability of retaining walls, the proposed nominal safety factors are significantly less than the current ones with more than 20% reduction. The comparison of allowable stresses indicates that the traditional safety provisions of the current WSD code are irrational and yield uneconomical designs in a number of cases, and thus have to be revised in order to confirm with the corresponding main LRFD code provisions.

Next, in order to check the consistency of the reliability of the proposed PBWSD according to the variation of the variable load ratios, β V.S. $\bar{L}/\bar{D}$, the curves are plotted for the PBWSD provisions with the corresponding LRFD provisions [3], as shown in Fig. 2. Figure 2 shows that the variation of the curves of the LRFD at β_0 are fairly insensitive to the variation of $\bar{L}/\bar{D}$, while those of the curves of the PBWSD are fairly sensitive, which is anticipated in the case of WSD code with single safety factor for each limit state. However, if we consider that the range of the variation of load ratio, $\bar{L}/\bar{D}$, for general R.C. building structures falls within 0.5-1.5, it can be seen that the deviations of the reliability indices, in most cases, are nothing but less that $\pm$ 0.2. This, also, indicates that the PBWSD provides practically consistent reliability-based design criteria.

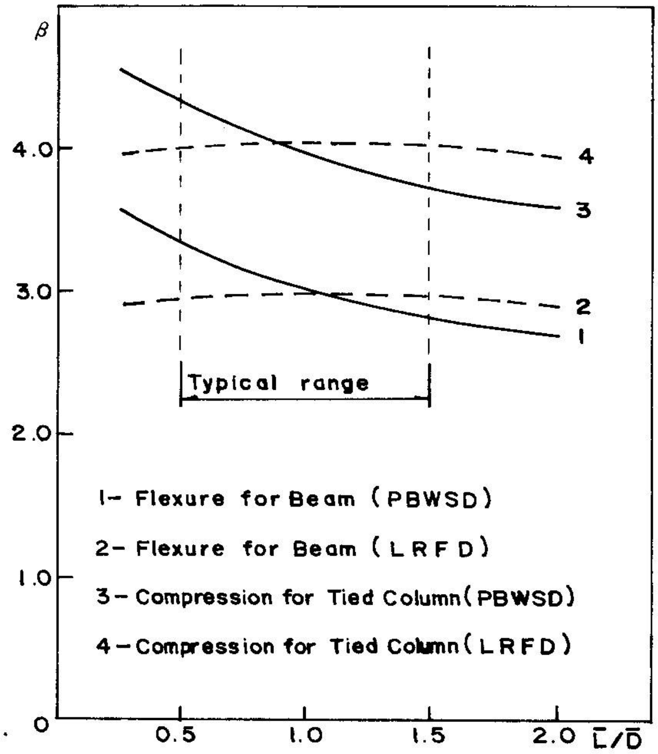

Fig. 2 Comparison of the Consistency of Reliability

5. CONCLUSIONS

The following conclusions can be drawn based on this study.

-1. The current WSD codes should be remodeled as the PBWSD by using the practical AFOSM reliability method and the simple code calibration procedure proposed in this study.

-2. Thus, the irrational allowable stresses of the current code can be replaced by a reasonably complex matrix of the rational allowable stresses which yields economical designs in a number of cases.

-3. The PBWSD can be used as an alternative design method for practitioners during the transition decade to come in Korea, which provides approximately as identical and consistent reliability as the corresponding primary LRFD code.

-4. More elaborate and systematic studies on the selection of optimal target reliability indices remain as further research area.

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