Zeitschrift: Elemente der Mathematik

Band: 1 (1946)

Heft: 1

Artikel: Present developments in the teaching of mathematics in the United States

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DOI: https://doi.org/10.5169/seals-1198

herzuleiten, die für $p=0$, $q=-1$; $k=l=1$; $f=\varphi$, $g=x$ die BERNOULLIsche Reihe (B) als Spezialfall enthalte. Den Beweis führt VARIGNON nach BERNOULLIS Manier, indem er von der allgemeineren Identität

$$\int^{p} f^{k} \int^{q} g^{l} = \left(\int^{p} f^{k} \int^{q} g^{l} + \int^{p-1} f^{k} \int^{q+1} g^{l}\right) - \left(\int^{p-1} f^{k} \int^{q+1} g^{l} + \int^{p-2} f^{k} \int^{q+2} g^{l}\right) + \cdots$$

ausgeht. Merkwürdigerweise hat VARIGNON diese Formel nie publiziert, wie wir von ihm ja überhaupt keine Arbeiten über den Infinitesimal*kalkül* als solchen besitzen, so sehr ihn auch seine geometrischen und mechanischen Arbeiten als einen Meister in der praktischen Handhabung desselben ausweisen.

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Present Developments in the Teaching of Mathematics in the United States

Two outstanding publications of the year 1945 characterize both the present general tendencies and the individual contributions in the development of the teaching of mathematics in the United States. Both publications have been brought out by committees, and each lists over a hundred co-workers. One is the report of the Harvard Committee, published by the President and the Fellows of Harvard College under the title "General Education in a Free Society". It gives a nation-wide survey of both school and collegiate education and also includes a special chapter on the teaching of mathematics. The Harvard Committee is composed of twelve members, of college presidents, deans and professors, and has been appointed by J. B. CONANT, President of Harvard University in the Spring of 1943. The following quotations may convey the direction of its findings: "The ideal is a system which shall be as fair to the fast as to the slow, to the hand-minded as to the book-minded, but which while meeting the separate needs of each, shall yet foster the fellow-feeling between human being and human being which is the deepest root of democracy ... Hence the task of the high-school is not merely to speed the bright boy to the top. It is at least as much so to widen the horizons of the ordinary student that they and, still more, their children will encounter fewer of the obstacles that cramp achievement ... A truly democratic education must perforce try to equalize opportunity by counteracting impediments. But it cannot do so simply by offering the conventional academic subjects to all students indiscriminately. These, again as now taught at least are too alien to the backgrounds of most students to be anything like generally effective in breaking down the barriers of circumstance. Something closer to their experience is needed which, by meeting them halfway will lead them out and beyond themselves."

In regard to mathematics in particular, the report points out the increased demands called forth by the necessities of modern industrial life and then stresses the general educational angle of the mathematical sciences. "The complexities of organization and technology in modern industry, in government, and in the national defense make increasing demands upon the mathematical equipment and skills of the ordinary participant and workman. The wartime situation in which many young men otherwise qualified for officer training were rejected because of deficiency in mathematics, can be duplicated in many varieties of employment. The fact is that there is a steadily increasing number of jobs in industry, as well as in both civil and governmental agencies, for which a sound training in algebra and geometry is a pre-requisite. For a fairly considerable number of positions solid geometry and trigonometry are essential. Beyond this, however, mathematics has an important intrinsic role in general education. It helps build some of the skills and comprehensions that make the effective individual. Within the past fifty years mathematics and logic have been fused into a single structure."

Answers to these general demands have been given specifically in the second publication, in the yearbook 1945 of the National Council of Teachers of Mathematics. It is published by the Bureau of Publications at Teachers' College Columbia University, New York. The title of the yearbook is "Multi-Sensory Aids in the Teaching of Mathematics". The National Council is composed of four officers and twelve additional members, selected from professors of teachers' colleges and leading schoolmen in twelve different states. The contributions have been edited by Professor W. D. REEVE in cooperation with a Committee whose members are spread over the United States from Spokane in the State of Washington in the farthest northwest, to the Atlantic Coast, under the chairmanship of Professor E. H. C. HILDEBRANDT of Northwestern University in Illinois.

The title of the Yearbook introduces a new term not yet found in the dictionaries which well describes its aims and content. From many realms of experience are the starting points taken to lead to mathematical conceptions. The tasks involved are two-fold. One part consists in building up as many devices and inventive suggestions as possible to enrich the approach to the mathematical sciences. The other has to do with the remodeling of the form of our knowledge, in such a manner, that the creative thought activity of the student will be stimulated. This holds good for all levels of educations and, as an example, it might be illustrated by the introduction of determinants. The sequence of concepts, as usually presented, leads from permutations, even and odd permutations, straight to determinants. Quoting from one of the college textbooks on the introduction to the theory of equations, the introduction of determinants proceeds as follows: "We propose now to explain the meaning of the symbolic array

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{vmatrix}$$

which is called a determinant of order n. The a's denote any numbers whatever, and are called the elements of the determinant. Any n elements which appear in the same horizontal line of the array are said to constitute a row of the determinant, and n elements in the same vertical line are said to constitute a column. Thus the first of the two subscripts associated with each of the a's indicates the row, and the second subscript denotes the column in which the element is found. These subscripts will be referred to a's row indices and column indices respectively. The determinant is used to denote a certain algebraic expression involving the a's. The explicit description of this function of the elements is embodied in the following definition: From all possible products of the a's taken n at a time, which can be obtained by permuting the second subscripts in the expression $a_{11}, a_{22}, a_{33}, a_{44}, \ldots, a_{nn}$ give each of these products a positive or a negative sign according as the second subscripts constitute an even or an odd permutation. The algebraic sum of these signed products is the expression which the determinant represents." From there the study proceeds to various concepts connected with determinants, as minors, co-factors, etc., then to the laws of handling determinants, as the expansion of determinants, LAPLACE's development, and finally, in a following chapter, to CRAMER's rule and the application of determinants to the solving of simultaneous linear equations.

This way represents a well-perfected route to the definition and use of determinants, but it confronts the student with a ready-made function which he has to accept by definition. He may even have the impression of an arbitrary choice. In fact determinants have never been found along these lines. Their relationship to permutations constitutes a secondary phenomenon. Determinants have been derived from the processes of solving simultaneous linear equations. This is the way, along which Chinese mathematicians first found their trace and it is also the way which LEIBNIZ followed in his research in 1693. Readjusting the order of presentation to the historic sequence, one will have to start with simultaneous equations and let the determinants evolve from the processes of their solution. This will make it possible for the students to understand the origin of determinants as well as it will give them the experience of re-inventing their concepts.

EUCLID's geometry on which we still base our introductions to plane geometry shows every characteristic of a final form, rather than one of a pioneer approach. EUCLID taught at Alexandria, the center of the last epoch of Greek culture which flourished after the Greek nation had already lost its independence. The systematic structure which we cherish in his books represents the ultimate form into which he merged the knowledge of the Greek mathematicians who had built it up during the three centuries from 600 to 300 B. C., from THALES and PYTHAGORAS down to his days. Keeping this in mind, we do not minimize its values but we should give them the appropriate place within our educational systems which will not be the one of the beginning. We should start instead with geometric constructions in a very much more elaborate manner than we do so far. Working with geometric drawing instruments, India ink, colors, etc. we can build up a foundation of experiences upon which later structures can be erected. A quotation from the article on "Geometric Drawing" in the Yearbook on Multi-Sensory Aids, by the author of this report reads, "Geometric drawing is a mathematical laboratory method to be compared with the laboratory work done in the natural sciences. It stimulates mathematical interest and serves as a basis for our other mathematics courses. Besides the opportunities which it offers in connection with mathematics, geometric drawing helps to develop manual skill. Many techniques are taught in the course. There is the handling of compasses, the drawing with triangles, the use of ruling pens and black ink, the application of color to both lines and areas, including shadow effects on solids and curved surfaces, and, finally, lettering. Through these techniques such artistic qualities as a sense of proportion when placing figures and script into a given space and skill in combining colors are developed . . . Geometric drawing thus holds a middle position between the academic work in the school and the arts and crafts, and offers special opportunities within the general educational tasks. It can be applied in various forms to different school levels." A sequence of examples from fundamental constructions of regular polygons, etc. leads to linear perspective and to the construction of curves and families of curves. From there, one can continue either to descriptive geometry or to analytic geometry.

What is particularly pleasant to experience within the United States is a spirit of live initiative and of friendly cooperation. The conventions of the National Council of Teachers of Mathematics which were held during the last years at Baton Rouge, Atlantic City, Boston, Bethlehem, San Francisco, and Denver, resulted in many impulses for suggestions and experimenting which hold numerous promising potentials for the future.

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Kleine Mitteilung

Der Satz «Jede Ebene, die nicht parallel zur Achse eines Rotationsparaboloids ist, schneidet dieses in einer Ellipse, deren senkrechte Projektion auf eine Normalebene zur Achse ein Kreis ist» läßt sich folgendermaßen leicht und elementar beweisen:

Eine Kugel gehe durch zwei Punkte verschiedener Kote A und B und berühre die Projektionsebene Π. Sei S der Spurpunkt der Geraden durch A und B, und T der Berührungspunkt. Dann gilt

$$\overline{ST} = \sqrt{\overline{SA}\cdot\overline{SB}} = \text{konst.}$$

Der geometrische Ort für T ist also ein Kreis um S. Der Ort für den Kugelmittelpunkt M ist als Schnittfigur der geraden Zylinderfläche über diesem Kreis und der Mittelnormalebene zu $\overline{AB}$, die nach Voraussetzung nicht parallel zur Zylinderachse ist, eine Ellipse.

Andererseits erhält man den geometrischen Ort für M durch Schneiden des Rotationsparaboloids, dessen Leitebene Π und dessen Brennpunkt z.B. A ist, mit der Mittelnormalebene von $\overline{AB}$.

Aus den beiden Überlegungen folgt unmittelbar der behauptete Satz.

W. LÜSSY

Aufgaben

1. Ein reguläres Fünfeck zu zeichnen, dessen Seiten als gerade Linien (also eventuell in ihrer Verlängerung) der Reihe nach durch fünf in der Ebene gegebene Punkte hindurchgehen. Wann ist die Aufgabe lösbar? P. FINSLER

2. Von einer Ellipse kennt man zwei Punkte, den Mittelpunkt und die Länge der großen Hauptachse. Es ist eine planimetrische Konstruktion der Hauptachsen verlangt. W. LÜSSY