Zeitschrift: Elemente der Mathematik

Herausgeber: Schweizerische Mathematische Gesellschaft

Band: 74 (2019)

Heft: 2

Artikel: A short proof of Morley's theorem

Autor: Pamfilos, Paris

DOI: https://doi.org/10.5169/seals-869235

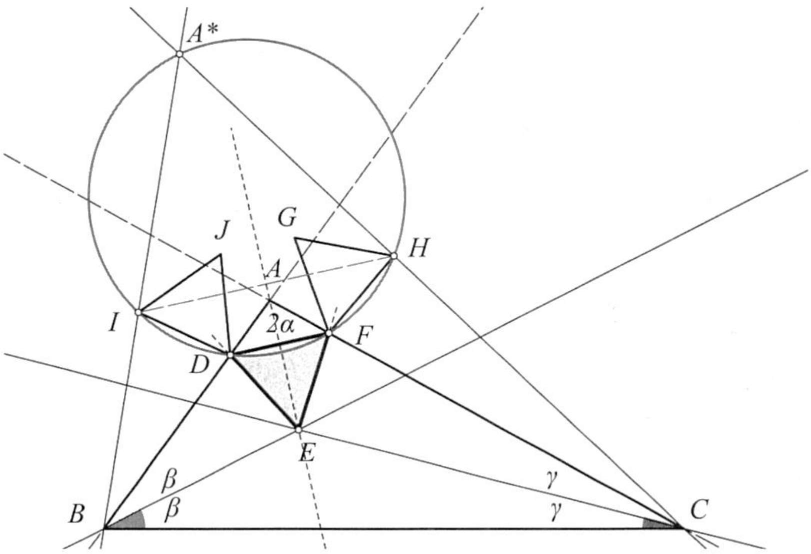
Elem. Math. 74 (2019) 80 – 81
0013-6018/19/020080-2
DOI 10.4171/EM/384

Elemente der Mathematik

Short note A short proof of Morley's theorem

Paris Pamfilos

Among the dozens of proofs of Morley's Theorem, the proofs of J.M. Child [J.M23] and Naraniengar [HC67, p. 47] are particularly neat. In this paper, we simplify them by stressing the symmetry of the equilateral and the symmetries created by two reflections.[1]



Bisector property

For this, we start with a trivial property of the bisectors of a triangle ABC with angles $\{2\alpha, 2\beta, 2\gamma\}$.

Lemma. *From the incenter E of triangle ABC and on both sides of AE draw two lines inclined to it by $30°$ and intersecting the other sides, respectively at D, F. Then DEF is equilateral and DF is orthogonal to AE.*

Having that, reflect BE and DEF on AB to obtain BI and the equilateral DIJ. Do the same on the other side, i.e., reflect CE and DEF on CF to obtain CH and the equilateral FHG. By the symmetry with respect to AE the quadrilateral $IDFH$ is an isosceles

[1] I would like to express my gratitude to the referee, for pointing out to me the proof by J.M. Child and making several useful remarks, helping me towards a better and more concise exposition.

trapezium with three equal sides, hence cyclic, and its angle $\widehat{IHF}$ is easily computed:

$$\widehat{IDF} = \widehat{IDJ} + \widehat{JDF} = 60^\circ + 2(90^\circ - \alpha) = 60^\circ + 180^\circ - 2\alpha = 60^\circ + 2\beta + 2\gamma \quad \Rightarrow$$
$$\widehat{IHF} = 180^\circ - \widehat{IDF} = 120^\circ - 2\beta - 2\gamma.$$

This, essentially, finishes the proof, since the arcs (IDF) and $(IDFH)$ are in ratio 2:3 and the angle at the intersection point A^* of lines BI and CH will be then of measure $\widehat{A^*} = 180^\circ - 3\beta - 3\gamma$. This shows that A^* is on the circlumcircle κ of $IDFH$ and $\{A^*D, A^*F\}$ are the trisectors of $\widehat{A^*}$, implying obviously that, *"for a triangle A^*BC with base angles $\{3\beta, 3\gamma\}$ the adjacent trisectors intersect at the vertices of an equilateral triangle"*, as required by the theorem of Morley.

References

[HC67] S. Greitzer, H. Coxeter. *Geometry Revisited.* Mathematical Association of America, Washington D.C., 1967.

[J.M23] J.M. Child. Proof of "Morley's theorem". *Mathematical Gazette*, 11:171, 1923.

Paris Pamfilos
University of Crete, Greece
e-mail: pamfilos@uoc.gr