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Rezensionen

D. Madan and W. Schoutens: Applied conic functions. 187 pages, £ 64.99. Cambridge University Press 2016, ISBN 978-1-107-15169-7 (hardback).

Classical financial mathematics has its origins in the problem of how to find fair prices of risky assets, that is, uncertain financial cash flows taking place at a future time T but traded today. One of its corner stones is the no-arbitrage principle. In non-technical terms, it says that it is almost impossible to make any gain at some future point in time without any starting capital and without taking any risk beforehand. Another basic topic of financial mathematics deals with the uniqueness of today's market prices of risky assets. Under certain natural assumptions prices are shown to be unique. Further, under these assumptions risk can be completely eliminated. The elimination of risk in the classical models which cannot be observed in the real financial markets is a fundamental aspect this book addresses.

Conic finance does not assume a unique price and does not completely exclude arbitrage. Although they might be rare, there are occasions of arbitrage opportunities. Moreover, in real-life financial markets the one-price assumption does not hold since differently from the ask price at which the market "is willing to sell" a given asset, there is the bid price at which the market "is willing to buy" this asset. Further, rather than trying to eliminate all risks (which is not possible) one should concede on acceptability of certain risks. This is where the subject of *risk measurement* enters into the picture as we shall see.

Let us assume that risky assets materializing at some future time T are modeled as finite random variables on some probability space $(\Omega, \mathcal{F}, P)$. For the sake of simplicity, Ω is assumed to be finite. Let X be a risky asset which the market is willing to buy today, i.e., at time zero, for the price c. Using the compound interest rate r, the risky asset $X - e^{rT}c$ has the price zero at time zero and thus constitutes a so-called *zero-cost cash flow*. Provided $X - e^{rT}c$ is positive, this zero cost cash flow is in fact an arbitrage opportunity.

The set $\mathscr{A}$ of risks materializing at T which the market *accepts at time zero* is modeled as a subset of L^1. Reasonably, this is assumed to be a convex cone: If $X, Y \in \mathscr{A}$ then $X + Y \in \mathscr{A}$ and if $Z \in \mathscr{A}$ then $\lambda Z \in \mathscr{A}$ for any $\lambda \geq 0$. One further requires that $\mathscr{A}$ intersects non trivially with the zero cost cash flows and should include at least the arbitrage opportunities among them.

Let $\mathcal{M}(\mathscr{A})$ be the set of probability measures Q on $\mathcal{F}$ such that $E^Q[X] \geq 0$ for all $X \in \mathscr{A}$. This is a convex subset in the space of all probability measures on $(\Omega, \mathcal{F})$. Intuitively, $\mathcal{M}(\mathscr{A})$ models the different attitudes present in the market towards acceptable risks. Now what is the best price at which the market is willing to buy X? This is given by

$$b(X) = \inf_{Q \in \mathcal{M}(\mathscr{A})} e^{-rT} E^Q[X],$$

the *bid price*. The best price at which the market is willing to sell X is given by

$$a(X) = \sup_{Q \in \mathcal{M}(\mathscr{A})} e^{-rT} E^Q[X],$$

the *ask price*. Now we have a frame work for a two-price economy, as $b(X) \leq a(X)$, where the inequality is usually strict. As detailed out in Chapter 4, the pricing functional a is nothing else than a *coherent risk measure on* $\mathscr{A}$.

One can argue, however, that the specification of the set of acceptable risks might be a daunting task. Fortunately, under additional assumptions on a, one can show that there is a concave function $\Psi : [0, 1] \to [0, 1]$, called a *probability distortion*, such that

$$a(X) = \int_{\mathbb{R}} x d\Psi(F_X(x)),$$

where F_X is the probability distribution of X. Since $b(X) = -a(-X)$, there is a similar expression for the bid price. Therefore, bid and ask prices can be specified by explicitly introducing a function Ψ as above. It is shown that in this two-price economy, ask and bid prices are no longer additive but subadditive and superadditive, that is, $a(X + Y) \leq a(X) + a(Y)$ and $b(X + Y) \geq b(X) + b(Y)$.

We have very briefly presented the rudiments of conic finance which is, as the title suggests, the central topic of the book. Conic finance starts with Chapter 4. In the remaining Chapters 5–11, the book offers numerous applications of conic finance: Among them, conic pricing of options, applications to risk measurement, portfolio optimization with respect to a diversity measure, conic hedging where the hedging result is measured in the introduced diversity measure (in a static and dynamic sense) and hedging of insurance contracts. The latter particularly shows the relevance of the theory developed to life insurance where the classical finance theory has its well-known limitations. The final chapter explains what is meant by trading in a Markovian context.

The book starts in a fairly elementary way by presenting the classical arbitrage theory and the famous Black Scholes valuation of European options. The reader acquainted with financial mathematics also might find a lot of interesting material in the introducing three chapters. We mention the variance-gamma processes and the Sato model in the context of financial mathematics. Variance-gamma processes are of interest here because they are pure jump processes in contrast to Brownian motion that is widely used as the source of stochasticity in stock values.

Despite the rich variety of topics covered, the book is comparably small. This is possible because most proofs of results are omitted. However, they can be found easily in the literature due to the extensive reference list.

Apart from carefully explaining all things developed, an emphasis is laid on detailed numerical examples which makes the book a very valuable introduction for beginners in the field of conic finance. It gives a new and fresh well-studied view on classical aspects on mathematical finance theory.

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Stephan Schultze, Küsnacht

Dirk W. Hoffmann: Die Gödel'schen Unvollständigkeitssätze. 356 Seiten, 26.99 € (Softcover). Springer, 2017; ISBN 978-3-662-54299-6.

Mit dem Untertitel *Eine geführte Reise durch Kurt Gödels historischen Beweis* präzisiert Hoffmann den Inhalt seines Buches: Es geht um eine Darstellung von Gödels Originalbeweis seiner Unvollständigkeitsresultate aus dem Jahr 1931.

Konsequenterweise beginnt das Buch mit einem Wegweiser in Form der 26 verkleinerten Faksimile Seiten von Gödels Originalarbeit und der Angabe, in welchem Abschnitt diese behandelt werden. Es folgt ein einleitendes Kapitel, in dem die Problematik der Unvollständigkeit von formalen Systemen erläutert wird. Zudem wird diese Fragestellung, sowie Gödels Arbeit in einen historischen Zusammenhang gestellt.

In den nächsten Kapiteln führt Hoffmann sein Programm zielorientiert durch: Die formalen Grundlagen der Mathematik werden erarbeitet und anschliessend eine Beweisskizze entworfen. Zentraler Punkt hierbei ist die Arithmetisierung der Syntax. Es folgt eine Einführung in das formale System P, sowie in die Theorie der primitiv rekursiven Funktionen und Relationen und deren Repräsentierbarkeit durch Formeln in P. Im abschliessenden Kapitel wird Gödels erster Unvollständigkeitssatz bewiesen und diskutiert. Zudem führt Hoffmann durch Gödels Beweisskizze des zweiten Unvollständigkeitssatzes.

Wie ein Leitmotiv werden immer wieder Abschnitte aus Gödels Originalarbeit eingestreut und danach erläutert und kommentiert. Dadurch gelingt es Hoffmann, die Materie auf eine sehr lebendige Weise zu vermitteln und dem Leser zudem einen tiefen Einblick in Gödels Ideen zu gewähren.

Da Hoffmann die benötigten mathematischen Grundkenntnisse fast vollständig einführt, macht er Gödels Unvollständigkeitssätze einem grossen mathematisch interessierten Publikum zugänglich.

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