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$$q_u = -q_{2u} = \prod_{p/u} (1 \pm p) \, . \tag{73}$$

Comparing, on both sides, the coefficients of x^3 we have, since $S_1(u) = u^3 - u$,

$$L(\mu - 1)\sigma_3 = -U(\mu) - 2\sigma_1^2 U(\mu - 2) + \tag{74}$$
$$+ \frac{\sigma_1^3}{6} (L(\mu - 3) - L(\mu - 1)) \, .$$

From this formula, we can again express σ_3 by means of the functions $U(s)$, $L(s)$ and proceeding in the same way obtain for a general $\sigma_\varkappa$, interpreted as a *formal* Dirichlet series, expressions containing only $\sigma_1, \ldots, \sigma_{\varkappa-1}$. However, already the expression foı σ_3 becomes essentially more complicated than those of σ_1 and σ_2. We give here only the expression for the coefficient of $\frac{1}{p^\mu}$ for an odd prime number p in σ_3:

$$\frac{1}{6}\left((-1)^{\frac{p-1}{2}} p - 1\right)\left((-1)^{\frac{p-2}{2}} p - 5\right)\left((-1)^{\frac{p-1}{2}} p - 6\right) \, . \, ,$$

which is easily obtained from (74) and has been derived directly by Dr. J. C. P. Miller. It is easy to see that this is always divisible by 16.

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