Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 8 (1962)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: SURVEY OF COBORDISM THEORY

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Bibliographie

DOI: https://doi.org/10.5169/seals-37949

$\Omega_k = H_k(\mathscr{D}_o)$ to $H_k(\mathscr{C}_o)$. Thom, Rohlin and Švarč have shown that Pontrjagin classes can be defined for combinatorial manifolds. Therefore we have:

THEOREM 3′. — *The homomorphism $\Omega_k \to H_k(\mathscr{C}_o)$ has kernel zero.*

However examples show that this homomorphism is not onto. The reader is referred to [13, 18].

Another interesting possibility would be to look at the class of compact homology manifolds.

Returning to the differentiable case, interesting cobordism groups can be obtained by restricting the connectivities of the manifolds involved. As an extreme case we can consider only differentiable manifolds which are either homotopy spheres or homotopy cells. The resulting cobordism groups are closely related to the problem of classifying differentiable structures on spheres. The reader is referred to Milnor [8] and Smale [14].

As a final, quite different, example consider differentiable imbeddings of the circle S^1 in the 3-sphere S^3. Such an object (a knot) is said to *bound* if it can be extended to a differentiable imbedding of the disk D^2 in the disk D^4. The resulting cobordism group has been studied by Fox and Milnor [5]. This group is not finitely generated.

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