A NEW FIXED POINT THEOREM FOR CONTINUOUS MAPS OF THE CLOSED n-CELL

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1. INTRODUCTION

In this paper the authors prove two fixed point theorems for continuous maps of a closed n-cell η^n into the euclidean space $R^n \supset \eta^n$. Neither theorem requires that η^n be mapped into itself.

The main theorem is Theorem 1 in which it is proved that *a continuous mapping of a closed n-cell* η^n *into* $R^n \supset \eta^n$ *which maps the boundary of* η^n *into* η^n, *has a fixed point.* It is believed that this theorem is new and is stronger than Brouwer's classical fixed point theorem inasmuch as it implies the latter and has weaker hypotheses.

Although the same theorem can be proved in a much shorter way by using Tietze's extension theorem followed by the classical Brouwer's fixed point theorem, however, in the proofs given below no knowledge of these two theorems is presupposed.

In this paper the proofs of the theorems are based in part on use of homologies, and in part on the turning index (defined below), which is essentially a generalization to the n dimensional case of the idea involved in [1], pp. 251-5, for the case of a circular disc.

2. NOTATION

In what follows, R^n denotes an oriented euclidean n-space, fixed once and for all.

All closed solid n-spheres and $(n-1)$-spheres are assumed to be triangulated with solid n-spheres oriented to agree with

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