

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 8 (1962)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** NEW FIXED POINT THEOREM FOR CONTINUOUS MAPS OF THE CLOSED  $n$ -CELL  
**Autor:** Abian, Alexander / Brown, Arthur B.  
**Kapitel:** 1. Introduction  
**DOI:** <https://doi.org/10.5169/seals-37951>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

**Download PDF:** 02.04.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

# A NEW FIXED POINT THEOREM FOR CONTINUOUS MAPS OF THE CLOSED $n$ -CELL

by ALEXANDER ABIAN <sup>1)</sup> AND ARTHUR B. BROWN

## 1. INTRODUCTION

In this paper the authors prove two fixed point theorems for continuous maps of a closed  $n$ -cell  $\eta^n$  into the euclidean space  $R^n \supset \eta^n$ . Neither theorem requires that  $\eta^n$  be mapped into itself.

The main theorem is Theorem 1 in which it is proved that *a continuous mapping of a closed  $n$ -cell  $\eta^n$  into  $R^n \supset \eta^n$  which maps the boundary of  $\eta^n$  into  $\eta^n$ , has a fixed point.* It is believed that this theorem is new and is stronger than Brouwer's classical fixed point theorem inasmuch as it implies the latter and has weaker hypotheses.

Although the same theorem can be proved in a much shorter way by using Tietze's extension theorem followed by the classical Brouwer's fixed point theorem, however, in the proofs given below no knowledge of these two theorems is presupposed.

In this paper the proofs of the theorems are based in part on use of homologies, and in part on the turning index (defined below), which is essentially a generalization to the  $n$  dimensional case of the idea involved in [1], pp. 251-5, for the case of a circular disc.

## 2. NOTATION

In what follows,  $R^n$  denotes an oriented euclidean  $n$ -space, fixed once and for all.

All closed solid  $n$ -spheres and  $(n-1)$ -spheres are assumed to be triangulated with solid  $n$ -spheres oriented to agree with

---

<sup>1)</sup> Formerly Smbat Abian The research of this author was supported in part by the National Science Foundation Grant G 17904.