

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 8 (1962)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: NEW FIXED POINT THEOREM FOR CONTINUOUS MAPS OF THE CLOSED n -CELL
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Kapitel: 3. The Turning Index
DOI: <https://doi.org/10.5169/seals-37951>

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R^n , and $(n-1)$ -spheres oriented with orientations induced by their interiors.

Symbols c^{n-1} , g^{n-1} , ... denote oriented $(n-1)$ -cycles in R^n ; D^{n-1} , V^{n-1} , ... denote $(n-1)$ -spheres in R^n . E^n denotes a closed solid n -sphere in R^n , and the boundary of E^n is denoted by S^{n-1} . η^n denotes a closed n -cell in R^n and the boundary of η^n is denoted by σ^{n-1} .

In this paper η^n is assumed to be the image of E^n under homeomorphism θ , and η^n and σ^{n-1} obtain their orientations from E^n and S^{n-1} respectively.

3. THE TURNING INDEX

Let c^{n-1} be an $(n-1)$ -cycle in R^n and g a continuous map of c^{n-1} into R^n having no fixed point. Let D^{n-1} be an $(n-1)$ -sphere with center 0, called a *direction sphere* [2]. Let c^{n-1} be mapped on D^{n-1} as follows. To a point $c \in c^{n-1}$ there corresponds a point $d \in D^{n-1}$ such that the line segment from 0 to d has the same sense and direction as that from c to $g(c)$. The resulting $(n-1)$ -cycle g^{n-1} on D^{n-1} is called, in the sequel, *the $(n-1)$ -cycle g^{n-1} resulting from g applied to c^{n-1}* , and the degree of the resulting map, that is, the multiple of D^{n-1} which is homologous to g^{n-1} (which is clearly independent of the radius of D^{n-1} and the location of 0) is called the *turning index* of c^{n-1} under g .

If p is a point not on c^{n-1} , the *index of p relative to c^{n-1}* is defined as the turning index of the map which maps every point of c^{n-1} into p . (For odd n , this is the negative of the corresponding definition given in [3], as shown by Theorem 1.5, page 105).

4. PRELIMINARY LEMMAS

LEMMA 1. *Let g and h be two continuous maps into R^n of an $(n-1)$ -cycle c^{n-1} , such that neither leaves any point of c^{n-1} fixed, and, for no point $c \in c^{n-1}$ are the directions from c to $g(c)$ and from c to $h(c)$ exactly opposite. Then the turning indices of c^{n-1} under g and h are equal.*

Proof. For each $c \in c^{n-1}$, the directions of the two vectors $c, g(c)$ and $c, h(c)$ are not opposite and hence, if not identical,