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Artikel: NEW FIXED POINT THEOREM FOR CONTINUOUS MAPS OF THE CLOSED n-CELL

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Kapitel: 3. The Turning Index

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R^n, and $(n-1)$-spheres oriented with orientations induced by their interiors.

Symbols c^{n-1}, g^{n-1}, ... denote oriented $(n-1)$-cycles in R^n; D^{n-1}, V^{n-1}, ... denote $(n-1)$-spheres in R^n. E^n denotes a closed solid n-sphere in R^n, and the boundary of E^n is denoted by S^{n-1}. η^n denotes a closed n-cell in R^n and the boundary of η^n is denoted by σ^{n-1}.

In this paper η^n is assumed to be the image of E^n under homeomorphism θ, and η^n and σ^{n-1} obtain their orientations from E^n and S^{n-1} respectively.

3. The Turning Index

Let c^{n-1} be an $(n-1)$-cycle in R^n and g a continuous map of c^{n-1} into R^n having no fixed point. Let D^{n-1} be an $(n-1)$-sphere with center 0, called a *direction sphere* [2]. Let c^{n-1} be mapped on D^{n-1} as follows. To a point $c \in c^{n-1}$ there corresponds a point $d \in D^{n-1}$ such that the line segment from 0 to d has the same sense and direction as that from c to $g(c)$. The resulting $(n-1)$-cycle g^{n-1} on D^{n-1} is called, in the sequel, *the $(n-1)$-cycle g^{n-1} resulting from g applied to c^{n-1}*, and the degree of the resulting map, that is, the multiple of D^{n-1} which is homologous to g^{n-1} (which is clearly independent of the radius of D^{n-1} and the location of 0) is called the *turning index* of c^{n-1} under g.

If p is a point not on c^{n-1}, the *index of p relative to c^{n-1}* is defined as the turning index of the map which maps every point of c^{n-1} into p. (For odd n, this is the negative of the corresponding definition given in [3], as shown by Theorem 1.5, page 105).

4. Preliminary Lemmas

Lemma 1. *Let g and h be two continuous maps into R^n of an $(n-1)$-cycle c^{n-1}, such that neither leaves any point of c^{n-1} fixed, and, for no point $c \in c^{n-1}$ are the directions from c to $g(c)$ and from c to $h(c)$ exactly opposite. Then the turning indices of c^{n-1} under g and h are equal.*

Proof. For each $c \in c^{n-1}$, the directions of the two vectors $\overline{c,g(c)}$ and $\overline{c,h(c)}$ are not opposite and hence, if not identical,