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In short, the turning index of  $\sigma^{n-1}$  under the assumption of the absence of fixed points is non-zero, a fact which contradicts Lemma 4. Hence  $f$  has at least one fixed point, and Theorem 2 is proved.

**COROLLARY 1.** *Let  $E^n$  be a closed solid  $n$ -sphere and  $f$  a continuous mapping of  $E^n$  into  $R^n$  such that  $f$  maps the boundary  $S^{n-1}$  of  $E^n$  into  $E^n$ . Then  $f$  has at least one fixed point.*

*Proof.* If no point of  $S^{n-1}$  is fixed, then the hypotheses of Theorem 2 are seen to be satisfied with  $e$  at the center of the sphere  $E^n$  and  $\alpha = 0$ .

Clearly, Corollary 1 also follows immediately from Theorem 1. Proofs of this corollary also appear in the literature ([3], page 115).

**COROLLARY 2.** *Let  $\eta^n \subset R^n$  be a closed  $n$ -cell with boundary  $\sigma^{n-1}$ , and  $f$  and  $g$  two continuous maps of  $\eta^n$  into  $R^n$  such that for no point  $\sigma \in \sigma^{n-1}$  is  $f(\sigma) = g(\sigma)$ . If there exists an inner point  $e$  of  $\eta^n$  and a constant angle  $\beta$ ,  $0 \leq \beta \leq \pi$ , such that for no point  $\sigma \in \sigma^{n-1}$  is  $\beta$  an angle between the vectors  $e, \sigma$  and  $f(\sigma), g(\sigma)$ , then there is a point  $\eta_0 \in \eta^n$  such that  $f(\eta_0) = g(\eta_0)$ .*

*Proof.* Consider the map  $h$  of  $\eta^n$  into  $R^n$  such that for every point  $\eta \in \eta^n$  the vectors  $\eta, h(\eta)$  and  $f(\eta), g(\eta)$  are equal. By Theorem 2, the map  $h$  has a fixed point  $\eta_0$ . Consequently,  $f(\eta_0) = g(\eta_0)$ .

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