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Artikel: NEW FIXED POINT THEOREM FOR CONTINUOUS MAPS OF THE CLOSED n-CELL

Autor: Abian, Alexander / Brown, Arthur B.

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In short, the turning index of σ^{n-1} under the assumption of the absence of fixed points is non-zero, a fact which contradicts Lemma 4. Hence f has at least one fixed point, and Theorem 2 is proved.

COROLLARY 1. *Let E^n be a closed solid n-sphere and f a continuous mapping of E^n into R^n such that f maps the boundary S^{n-1} of E^n into E^n. Then f has at least one fixed point.*

Proof. If no point of S^{n-1} is fixed, then the hypotheses of Theorem 2 are seen to be satisfied with e at the center of the sphere E^n and $\alpha = 0$.

Clearly, Corollary 1 also follows immediately from Theorem 1. Proofs of this corollary also appear in the literature ([3], page 115).

COROLLARY 2. *Let $\eta^n \subset R^n$ be a closed n-cell with boundary σ^{n-1}, and f and g two continuous maps of η^n into R^n such that for no point $\sigma \varepsilon \sigma^{n-1}$ is $f(\sigma) = g(\sigma)$. If there exists an inner point e of η^n and a constant angle β, $0 \leqq \beta \leqq \pi$, such that for no point $\sigma \varepsilon \sigma^{n-1}$ is β an angle between the vectors $\overline{e, \sigma}$ and $\overline{f(\sigma), g(\sigma)}$, then there is a point $\eta_0 \varepsilon \eta^n$ such that $f(\eta_0) = g(\eta_0)$.*

Proof. Consider the map h of η^n into R^n such that for every point $\eta \varepsilon \eta^n$ the vectors $\overline{\eta, h(\eta)}$ and $\overline{f(\eta), g(\eta)}$ are equal. By Theorem 2, the map h has a fixed point η_0. Consequently, $f(\eta_0) = g(\eta_0)$.

REFERENCES

1. COURANT, R. and ROBBINS, R. E., *What is Mathematics ?*, Oxford, 1941.
2. L. E. J. BROUWER, Über Abbildung von Mannigfältigkeiten, *Math. Annalen*, Vol. 71 (1912), pp. 97-115.
3. P. S. ALEKSANDROV, *Combinatorial Topology*, Vol. 3, Graylock Press, Albany, N.Y., 1960.
4. L. S. PONTRYAGIN, *Foundations of Combinatorial Topology*, Graylock Press, Rochester, N.Y., 1952.

University of Pennsylvania
and
Queens College of the City University of New York.