Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 8 (1962)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON THE CONSTRUCTION OF RELATED EQUATIONS FOR THE ASYMPTOTIC THEORY OF LINEAR ORDINARY DIFFERENTIAL EQUATIONS ABOUT A TURNING POINT

Autor: Langer, Rudolph E.

Bibliographie

DOI: https://doi.org/10.5169/seals-37963

The replacements which change T to T_ν are thus seen to be ones which replace

$$\lambda^{n-\nu}\left\{ \delta_{p-\nu, j} + \frac{\sigma_{j,r}^{(p-\nu)}}{\lambda^r} \right\} \quad \text{by} \quad \lambda^n \frac{\tau_\nu}{\lambda^r}.$$

It follows that

$$\frac{T_\nu}{T} = \lambda^\nu \frac{\theta_\nu(z, \lambda)}{\lambda^r},$$

with some function $\theta_\nu(z, \lambda)$ which is bounded over the z and λ domains. This gives to the relation (9. 3) the form

$$L^*(u) = l^*(m^*(u)) - \frac{1}{\lambda^r} \sum_{\nu=1}^{p} \lambda^\nu \theta_\nu D^{p-\nu} m^*(u). \quad (9.7)$$

With the substitution of the expression for $D^{p-\nu} m^*(u)$, as it may be obtained from (4. 3) by writing $\bar{\gamma}_{i-s}$ in the place of γ_{i-s}, it is found that

$$L^*(u) = l^*(m^*(u)) - \frac{1}{\lambda^r} \sum_{j=1}^{n} \lambda^j \omega_j(z, \lambda) D^{n-j} u, \quad (9.8)$$

with

$$\omega_j(z, \lambda) = \sum_{\nu=1}^{p} \sum_{s=0}^{p} \lambda^{-s} \binom{p-\nu}{s} \theta_\nu D^s \bar{\gamma}_{\mu-\nu-s}.$$

A comparison of this with the earlier result (6. 6) shows that

$$L^*(u) = L(u) - \frac{1}{\lambda^r} \sum_{j=1}^{n} \lambda^j \{ \varepsilon_j(z, \lambda) + \omega_j(z, \lambda) \} D^{n-j} u. \quad (9.9)$$

The equation (9. 1), whose solutions are completely known, thus has coefficients which differ from those of the given equation (2. 1) only by terms that are of at least the r^{th} degree in $1/\lambda$. It is, therefore, by definition, a related equation.

REFERENCES

[1] R. E. LANGER, The asymptotic solutions of ordinary linear differential equations of the second order with special reference to a turning point. *Trans. Amer. Math. Soc.*, vol. 67 (1949), pp. 461-490.

[2] R. W. MCKELVEY, The solutions of second order linear ordinary differential equations about a turning point of order two. *Trans. Amer. Math. Soc.*, vol. 79 (1955), pp. 103-123.

[3] R. E. LANGER, The solutions of a class of ordinary differential equations of the third order in a region containing a multiple turning point. *Duke Math. Jour.*, vol. 23 (1956), pp. 93-110.
[4] N. D. KAZARINOFF, Asymptotic theory of second order differential equations with two simple turning points. *Archive f. Rat. Mech. and Anal.*, vol. 2 (1958), pp. 129-150.
[5] R. E. LANGER, On the construction of related differential equations. *Trans. Amer. Math. Soc.*, vol. 81 (1956), pp. 394-410.
[6] R. E. LANGER, Formal solutions and a related equation for a class of fourth order differential equations of hydrodynamic type. *Trans. Amer. Math. Soc.*, vol. 92 (1959), pp. 371-410.

Mathematics Research Center
U.S. Army.
University of Wisconsin
Madison, Wis.
U.S.A.