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# VERTEX POINTS OF FUNCTIONS

by Ali R. AMIR-MOÉZ

For  $f$  a real function of  $n$  variables, usually the Hessian matrix is studied in connection with Gaussian and mean curvatures of  $f(x_1, \dots, x_n)$ . In this paper we study other properties of  $f$  in a neighborhood of a point. In particular we get a method for obtaining vertex points of the function  $f$ . We also generalize the idea to some complex cases.

## 1. DEFINITIONS AND NOTATIONS

Let  $f$  a function of complex variables  $x_1, \dots, x_n$  be of class  $C''$  in  $x_1, \dots, x_n$ , and  $\bar{x}_1, \dots, \bar{x}_n$ , in a neighborhood of a point. Then  $f$  is called unitarily analytic if

$$\frac{\partial^2 f}{\partial x_i \partial \bar{x}_j} = \overline{\left( \frac{\partial^2 f}{\partial \bar{x}_i \partial x_j} \right)}.$$

*Theorem:* Let  $f$  be of class  $C''$  in  $x_1, \dots, x_n, \bar{x}_1, \dots, \bar{x}_n$  in a neighborhood of a point, and

$$\frac{\partial f}{\partial \bar{x}_k} = \overline{\left( \frac{\partial f}{\partial x_k} \right)}.$$

Then  $f$  is unitarily analytic.

The proof is quite simple and we omit it. Note that the converse is not necessarily true.

## 2. TANGENT QUADRIC

Let  $f$  be unitarily analytic in a neighborhood of  $(c_1, \dots, c_n)$ .

Let, for example,  $\frac{\partial f}{\partial c_1}$  be the value of  $\frac{\partial f}{\partial x_1}$  at  $(c_1, \dots, c_n)$ , and