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VERTEX POINTS OF FUNCTIONS

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For f a real function of n variables, usually the Hessian matrix is studied in connection with Gaussian and mean curvatures of $f(x_1, \ldots, x_n)$. In this paper we study other properties of f in a neighborhood of a point. In particular we get a method for obtaining vertex points of the function f. We also generalize the idea to some complex cases.

1. Definitions and notations

Let f a function of complex variables $x_1, \ldots, x_n$ be of class C'' in $x_1, \ldots, x_n$, and $\bar{x}_1, \ldots, \bar{x}_n$, in a neighborhood of a point. Then f is called unitarily analytic if

$$\frac{\partial^2 f}{\partial x_i \, \partial \bar{x}_j} = \left(\overline{\frac{\partial^2 f}{\partial \bar{x}_i \, \partial x_j}} \right).$$

Theorem: Let f be of class C'' in $x_1, \ldots, x_n, \bar{x}_1, \ldots, \bar{x}_n$ in a neighborhood of a point, and

$$\frac{\partial f}{\partial \bar{x}_k} = \left(\overline{\frac{\partial f}{\partial x_k}} \right).$$

Then f is unitarily analytic.

The proof is quite simple and we omit it. Note that the converse is not necessarily true.

2. Tangent Quadric

Let f be unitarily analytic in a neighborhood of $(c_1, \ldots, c_n)$. Let, for example, $\dfrac{\partial f}{\partial c_1}$ be the value of $\dfrac{\partial f}{\partial x_1}$ at $(c_1, \ldots, c_n)$, and