

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 10 (1964)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: VERTEX POINTS OF FUNCTIONS
Autor: Amir-Moéz, Ali R.
Kapitel: 5. Vertex points
DOI: <https://doi.org/10.5169/seals-39423>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 02.04.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

II. Let the rank of Q be k , and centers exist. Then these centers are solutions of

$$\xi_k = \xi E = - \left(\frac{\partial f}{\partial \xi} \right) EQ^{-1}, \quad (3.2)$$

where Q^{-1} is the reciprocal of Q , see [2]. That is, if E is the projection on the range of Q , then

$$Q^{-1} Q = QQ^{-1} = E.$$

Here we choose the center of quadric curvatures at a point of (3.2) so that, it is at the shortest distance from γ .

III. When the rank of Q is k and the quadric does not have centers, then we say that f does not have a center of quadric curvature.

4. DIRECTION OF QUADRIC CURVATURE

In part I and II of section 3 we respectively call the vectors ξ and ξ_k the directions of quadric curvature of f at (c_1, \dots, c_n) . In III of section 3, we define the direction of quadric curvature to be a vector δ which satisfies

$$\delta = \delta E = - \left(\frac{\partial f}{\partial \xi} \right) EQ^{-1},$$

where E is the projection described in section 3.

5. VERTEX POINTS

Let at the point $\gamma = (c_1, \dots, c_n)$ of f the direction of quadric curvature be the same as the normal to $f = 0$. Then γ is called a vertex point of the function f .

Theorem: A necessary and sufficient condition for a point to be a vertex point of the function f is that at that point

$$PQ = QP,$$

where P and Q are the matrices described in section 3.

Proof: At a vertex point the projection of the direction of quadric curvature on the tangent plane is zero. Thus

$$-\left(\frac{\partial f}{\partial \xi}\right) Q^{-1} (I - P) = 0.$$

This implies that

$$Q^{-1} P Q = P.$$

In all cases this implies

$$P Q = Q P.$$

A vertex point in particular may become a spherical point, i.e. a point where

$$\frac{\partial^2 f}{\partial x_i \partial \bar{x}_j} = \lambda \delta_{ij}, \lambda$$

is a constant.

A vertex point will be called a cylindrical point when

$$\frac{\partial^2 f}{\partial x_i \partial \bar{x}_j} = \lambda \delta_{ij}, i, j \leq k,$$

$$\frac{\partial^2 f}{\partial x_i \partial \bar{x}_j} = 0, i, j > k.$$

6. FUNCTIONS OF FIXED CENTER

An interesting fact about these functions is that they are not necessarily quadrics.

The equation.

$$\xi Q = -\left(\frac{\partial f}{\partial \xi}\right) \tag{6.1}$$

where $\xi = (c_1 - x_1, \dots, c_n - x_n)$, and (c_1, \dots, c_n) is the fixed center gives f . To produce a counter example we let the origin be the center and the dimension of the space be two. Then in the real case (6.1) becomes