

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 14 (1968)
Heft: 1: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ANALYTIC SPACES
Autor: Malgrange, Bernard
Kapitel: 4.4. The finiteness theorem
DOI: <https://doi.org/10.5169/seals-42341>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 13.03.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

Remark. If X is not separated, an intersection of two open Stein subspaces of X need not be Stein; take f.i. for X two copies of \mathbb{C}^2 , identified everywhere except at O ; there is an obvious covering of X by two open subspaces, identicals with \mathbb{C}^2 ; but their intersection is $\mathbb{C}^2 - \{O\}$, and therefore is not Stein!

4.4. The finiteness theorem

Theorem 4.4.1. (Cartan — Serre). Let X be a compact analytic space, and F be a coherent analytic sheaf on X . Then, for every $p \geq 0$ $H^p(X, F)$ is separated and finite dimensional.

We shall give two proofs of this theorem ; both are interesting for further applications.

1st proof. Let $\{X_i\}$ and $\{X'_i\}$ be two finite coverings of X of the type considered in the previous articles, such that, for every i , X'_i is relatively compact in X_i . Then, if we denote by \mathcal{U} (resp. \mathcal{U}') the covering $\{X_i\}$ (resp. $\{X'_i\}$), the natural restriction map $C^p(\mathcal{U}, F) \rightarrow C^p(\mathcal{U}', F)$ is compact.

Consider now the map

$$(\rho, d) : Z^p(\mathcal{U}, F) \oplus C^{p-1}(\mathcal{U}', F) \rightarrow Z^p(\mathcal{U}', F)$$

this map is surjective, and we have $(, d\rho) = (\rho, 0) + (0, d)$, $(\rho, 0)$ being compact ; then the following lemma proves that $\text{Im}(0, d)$ is closed and finite codimensional, q.e.d.

Lemma 4.4.2. Let E and F two Frechet spaces, u_1 and u_2 two linear continuous maps $E \rightarrow F$ such that $u_1 + u_2$ is surjective, and u_1 compact. Then $\text{Im}(u_2)$ is closed and finite codimensional. For the proof, see e.g. [5].

2nd proof. Consider \mathcal{U} and \mathcal{U}' as above, and consider the map $(\rho, d) : C^{p-1}(\mathcal{U}, F)/Z^{p-1}(\mathcal{U}, F) \rightarrow [C^{p-1}(\mathcal{U}', F)/Z^{p-1}(\mathcal{U}', F)] \oplus Z^p(\mathcal{U}, F)$ (ρ, d) is clearly injective. I claim that its image is closed: In fact, since $\bar{\rho} : H^p(\mathcal{U}, F) \rightarrow H^p(\mathcal{U}', F)$ is injective, this image consists of the pairs (\bar{a}', b) , $a' \in C^{p-1}(\mathcal{U}', F)$, $b \in Z^p(\mathcal{U}, F)$ such that $da' = \rho b$, which proves the assertion.

Now we have $(\rho, d) = (\rho, 0) + (0, d)$ and $(\rho, 0)$ is compact. By a well-known lemma, it results that $\text{Im}(0, d)$ is closed, which means that $H^p(\mathcal{U}, F)$ is separated.

Finally, since $\bar{\rho}$ is compact, and is an isomorphism, it follows that the identity map of $H^p(\mathcal{U}, F)$ into itself is compact ; therefore this space is finite dimensional ; this proves the theorem.