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Theorem (Krull). Let A be a local noetherian ring, E a finite module over A . Then:

- (i) The Krull topology of E is separated.
- (ii) Every submodule F of E is closed in E .
- (iii) The topology induced by E in a submodule F is the Krull topology of F .

Proof. (i) Let $F = \bigcap_{k \geq 0} m^k E = \overline{\{0\}}$. Then

$$mF = m((m^n E) \cap F) = (m^{n+1} E) \cap F = F$$

by the Artin-Rees lemma. Hence Nakayama's lemma implies that $F = \{0\}$.

(ii) Let $f: E \rightarrow E/F$ be the natural map. Then

$$f(\bar{F}) \subset f(F + m^k E) = f(m^k E) = m^k (E/F).$$

Hence $f(\bar{F}) \subset \bigcap m^k (E/F) = \{0\}$, using (i). But $f(\bar{F}) \subset \{0\}$ is equivalent to $\bar{F} \subset F$.

(iii) It is clear that $m^k F \subset (m^k E) \cap F$. Hence the Krull topology of F is finer than that induced by E ; in other words the inclusion $F \rightarrow E$ is continuous. Conversely the Artin-Rees lemma shows that

$$(m^{n+k} E) \cap F = m^k ((m^n E) \cap F) \subset m^k F$$

which proves that the induced topology is finer than the Krull topology of F .

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