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*Theorem* (Krull). Let $A$ be a local noetherian ring, $E$ a finite module over $A$. Then:

(i) The Krull topology of $E$ is separated.

(ii) Every submodule $F$ of $E$ is closed in $E$.

(iii) The topology induced by $E$ in a submodule $F$ is the Krull topology of $F$.

*Proof.* (i) Let $F = \bigcap_{k \geq 0} \mathfrak{m}^k E = \overline{\{0\}}$. Then

$$\mathfrak{m}F = \mathfrak{m}\big((\mathfrak{m}^n E) \cap F\big) = (\mathfrak{m}^{n+1} E) \cap F = F$$

by the Artin-Rees lemma. Hence Nakayama's lemma implies that $F = \{0\}$.

(ii) Let $f : E \to E/F$ be the natural map. Then

$$f(\bar{F}) \subset f(F + \mathfrak{m}^k E) = f(\mathfrak{m}^k E) = \mathfrak{m}^k (E/F) .$$

Hence $f(\bar{F}) \subset \cap\, \mathfrak{m}^k (E/F) = \{0\}$, using (i).). But $f(\bar{F}) \subset \{0\}$ is equivalent to $\bar{F} \subset F$.

(iii) It is clear that $\mathfrak{m}^k F \subset (\mathfrak{m}^k E) \cap F$. Hence the Krull topology of $F$ is finer than that induced by $E$ ; in other words the inclusion $F \to E$ is continuous. Conversely the Artin-Rees lemma shows that

$$(\mathfrak{m}^{n+k} E) \cap F = \mathfrak{m}^k \big((\mathfrak{m}^n E) \cap F\big) \subset \mathfrak{m}^k F$$

which proves that the induced topology is finer than the Krull topology of $F$.

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