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$$= \omega_{n-1} \int_0^\infty r^{n-1} f(r) \{ \int_{\Sigma_{n-1}} e^{-2\pi i |y| r(\eta \cdot \xi)} P^{(k)}(\xi.1)\, d\xi \}\, dr$$

Writing $y = t\,\eta$, this means that we have to compute

$$\int_{\Sigma_{n-1}} e^{-2\pi i r t(\eta \cdot \xi)} P^{(k)}(\xi.1)\, d\xi \,.$$

But, by the Funk-Hecke theorem (4.16) this integrãl is equal to

$$P^{(k)}(\eta.1)\, a_k^{-2}\, c_n \int_{-1}^{1} e^{-2\pi i r t s} P^{(k)}(s)(1-s^2)^{\frac{n-3}{2}}\, ds \,.$$

On the other hand, by (4.4), and, then integrating by parts k times we have

$$\int_{-1}^{1} e^{-2\pi i r t s} P^{(k)}(s)(1-s^2)^{\frac{n-3}{2}}\, ds = \alpha_{k,n} \int_{-}^{1} e^{-2\pi i r t s} \left[\frac{d^k}{dt^k} (1-s^2)^{k+\frac{n-3}{2}} \right] ds$$

$$= \beta_{k,n} \int_{-} (rt)^k e^{2\pi i r t s} (1-s^2)^{k+\frac{n-3}{2}}\, ds \,.$$

The last integral, however, is the one involved in the definition of J_λ when $\lambda = (2k+n-2)/2$. Equality (6.10) now follows immediately.[1])

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1) The Bessel functions we have encountered here arise in much the same way as did the ultraspherical Polynomials. Instead of the group $SO(n)$, however, one must study the group of all rigid motions on $\mathbf{R}^{(n)}$ (see Vilenkin [11] for details).

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