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*Corollary 13* Given the situation in Lemma 12, assume also that for some  $x_0 \in X(k)$ ,  $f(\{x_0\} \times Y)$  is the point  $z_0$ . Then  $f(X \times Y) = \{z_0\}$ .

*Proof:*

By the rigidity lemma, there exists  $g : Y \rightarrow Z$  such that  $f = g \circ p_2$ .  
 $f(x, y) = (g \circ p_2)(x, y) = g(y) = (g \circ p_2)(x_0, y) = f(x_0, y) = z_0$ .

*Corollary 14* If  $X$  and  $Y$  are abelian varieties and  $f : X \rightarrow Y$  is any morphism, then there exists a homomorphism  $h : X \rightarrow Y$  and a  $k$ -point  $a \in Y(k)$  such that  $f = T_a \circ h$  where  $T_a$  denotes translation by  $a$ .

*Corollary 15* Let  $X$  and  $Y$  be abelian varieties. Then  $X$  and  $Y$  are isomorphic as abelian varieties  $\Leftrightarrow X$  and  $Y$  are isomorphic as schemes.

*Proof:*

( $\Rightarrow$  .) obvious

( $\Leftarrow$  .) Let  $f : X \rightarrow Y$  be an isomorphism of schemes.  $f$  can be written as  $f = Y_a \circ h$  with  $a \in Y(k)$  and  $h$  a homomorphism.  $T_a$  is an isomorphism of schemes with  $T_{-a}$  as its inverse. Therefore  $h = T_{-a} \circ f$  is an isomorphism of schemes and hence of abelian varieties.

*Corollary 16* Let  $X$  be a variety and suppose that  $(X, m)$  and  $(X, m')$  are two abelian variety structures on  $X$  with identity elements  $e$  and  $e'$  respectively. Then  $m$  and  $m'$  differ only by translation.

*Proof:*

Let  $+$ ,  $-$ , and translation all denote operations with respect to  $m$ . Consider the morphism  $(m - m') : X \times X \rightarrow X$ . We have  $(m - m')(X \times \{e'\}) = e' = (m - m')(\{e'\} \times X)$ . By Corollary 13,  $(m - m')(X \times Y) = e'$ , i.e.  $m = m' + e'$ .

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