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### 8. CASE OF A MANIFOLD WITH BOUNDARY

More generally we consider a closed manifold  $N$  of dimension  $p$  in a manifold  $M$  of dimension  $n$ .  $L_{M,N}$  will denote the subalgebra of  $L_M$  of those vector fields on  $M$  which are tangent to  $N$ . An interesting particular case is when  $N$  is the boundary  $\partial M$  of  $M$ . For  $M$  compact,  $L_{M,\partial M}$  can be considered as the Lie algebra of the group of diffeomorphisms of  $M$ .

First we consider the formal vector fields. Let  $\mathfrak{a}_{n,p}$  be the Lie subalgebra of formal vector fields on  $R^n$  which are tangent to  $R^p$  identified to a linear subspace of  $R^n$ . Again  $C^*(\mathfrak{a}_{n,p})$  denotes the DG-algebra of those multilinear alternate forms on  $\mathfrak{a}_{n,p}$  depending only on finite order jets.

We describe a finite dimensional model for  $C^*(\mathfrak{a}_{n,p})$ . Let  $E(h'_1, \dots, h'_p, h''_1, \dots, h''_{n-p})$  be the exterior algebra in generators  $h'_i$  and  $h''_j$  of degree  $2i-1$ . Let  $R[c'_1, \dots, c'_p, c''_1, \dots, c''_{n-p}]_{2p}^\wedge$  be the quotient of the polynomial algebra in generators  $c'_i$  and  $c''_i$  of degree  $2i$  by the ideal of elements of degree  $> 2p$ .

Define

$$\begin{aligned} WU_{n,p} &= E(h'_1, \dots, h'_p, h''_1, \dots, h''_{n-p}) \\ &\otimes R[c'_1, \dots, c'_p, c''_1, \dots, c''_{n-p}]_{2p}^\wedge \end{aligned}$$

as the DG-algebra with differential defined by

$$dh'_i = c'_i, \quad dh''_i = c''_i, \quad dc'_i = 0, \quad dc''_i = 0.$$

This is a model for the space  $F_{n,p}$  obtained by restricting the universal principal  $(U_p \times U_{n-p})$ -bundle over the  $2p$ -skeleton of its basis represented by a product of Grassmanians with the usual even dimensional cell decomposition.

If  $n \leq 2p$ ,  $WU_{n,p}$  is also a model for a wedge of spheres. When  $n > 2p$ , it is a model for the product of the wedge of spheres corresponding to  $WU_{2p,p}$  by  $S^{2p+1} \times S^{2p+3} \dots \times S^{2n-2p-1}$ .

**THEOREM 1 (Koszul [11]).** *There is a natural morphism*

$$WU_{n,p} \rightarrow C^*(\mathfrak{a}_{n,p})$$

*inducing an isomorphism in cohomology.*

As a consequence,  $H^i(\mathfrak{a}_{n,p}) = 0$  for  $0 < i \leq 2p$  and  $i > p^2 + (n-p)^2 + 2p$ . When  $n \leq 2p$ , the multiplication is trivial.

To have a model for the homomorphism induced by the inclusion of  $\mathfrak{a}_{n,p}$  in  $\mathfrak{a}_n$ , we have the commutative diagramm

$$\begin{array}{ccc} C^*(\mathfrak{a}_n) & \longrightarrow & C^*(\mathfrak{a}_{n,p}) \\ \uparrow & & \uparrow \\ WU_n & \longleftarrow & WU_{n,p} \end{array}$$

where the second horizontal map sends  $h_i$  on  $h_i' + h_i''$  and  $c_i$  on  $c_i' + c_i''$  (by convention,  $h_i'$  or  $h_i''$  is zero for  $i > p$  or  $i > n-p$ , idem for  $c_i'$  and  $c_i''$ ). Note that the natural map of theorem 1 should map the  $c_i'$  s and  $c_i''$  not on the usual Chern classes defined by the connection but on the polynomials in Chern classes corresponding to  $\sum x_k^i$ , the Chern classes being the elementary symmetric functions in the formal variables  $x_k$ . These horizontal maps are also models for an inclusion of  $F_{n,p}$  in  $F_n$ .

We consider again the bundle  $E$  over  $M$  associated to the tangent bundle of  $M$  and with fiber  $F_n$ . Its restriction above  $N$  contains a subbundle  $E'$  with fiber  $F_{n,p}$ .

**THEOREM.**  $C^*(L_{M,N})$  is a model for the space  $\Gamma_{M,N}$  of continuous sections of the bundle  $E$  whose restriction to  $N$  have values in the subbundle  $E'$ .

To make explicit computations, we construct a model for  $\Gamma_{M,N}$ , which will be finite dimensional in each degree when  $M$  and  $N$  have finite dimensional models. This is the purpose of the next paragraph.

### 9. CONSTRUCTION OF A MODEL FOR $C^*(L_{M,N})$

Consider the commutative diagramm of Lie algebras

$$\begin{array}{ccc} L_{M,N} & \longrightarrow & L_M \\ \downarrow & & \downarrow \\ L'_{M,N} & \longrightarrow & L'_M \end{array}$$

where  $L'_M$  and  $L'_{M,N}$  are the quotients of  $L_M$  and  $L_{M,N}$  by the subalgebra  $L_M^0$  of vector fields on  $M$  whose infinite jet vanish at points of  $N$ .