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§1. PRELIMINARIES

Let G be a connected real semi-simple Lie group with finite center, K a maximal compact subgroup of G , and let $\mathfrak{g} \supset \mathfrak{k}$ be the corresponding Lie algebras. For any sub-algebra $\mathfrak{a} \subset \mathfrak{g}$, we put

$$\mathfrak{a}_{\mathbb{C}} = \mathfrak{a} \otimes_{\mathbb{R}} \mathbb{C}.$$

If B denotes the Killing form of \mathfrak{g} , B is negative-definite on \mathfrak{k} , and we let \mathfrak{p} denote the orthogonal complement under B of \mathfrak{k} in \mathfrak{g} . Then $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ is a so-called Cartan decomposition of \mathfrak{g} , and B is positive-definite on \mathfrak{p} .

Let $M = G/K$, the corresponding symmetric space. As $[\mathfrak{k}, \mathfrak{p}] \subset \mathfrak{p}$, B defines an $(\text{Ad } K)$ -invariant inner product on \mathfrak{p} ; and since we may identify \mathfrak{p} naturally as the tangent space to M at the identity coset $x_0 = K$, B determines a unique Riemannian metric on M which is invariant under the canonical left G -action.

Assume initially that M is an *irreducible* symmetric space. Then, if one wishes, G can be taken to be a non-compact almost simple group (i.e., \mathfrak{g} is a simple Lie algebra). In that case, the space M admits a homogeneous complex structure, and becomes an *Hermitian* symmetric space, precisely when \mathfrak{k} has a non-trivial center \mathfrak{z} . In this case, $\dim \mathfrak{z} = 1$, and $Z = \exp \mathfrak{z}$ is the identity component of the center of K . Let G^{ad} denote the adjoint group of G (i.e., the automorphism group of M) and let $K^{\text{ad}}, Z^{\text{ad}}$ be the corresponding subgroups of G^{ad} . A choice of $z_0 \in Z^{\text{ad}}$ of order 4 (for which $\text{Ad}(z_0^2)$ is a Cartan involution of \mathfrak{g}) determines an almost-complex structure on \mathfrak{p} :