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§1. PRELIMINARIES

Let G be a connected real semi-simple Lie group with finite center, K a maximal compact subgroup of G, and let $\mathfrak{g} \supset \mathfrak{k}$ be the corresponding Lie algebras. For any sub-algebra $\mathfrak{a} \subset \mathfrak{g}$, we put

$$\mathfrak{a}_{\mathbf{C}} = \mathfrak{a} \otimes_{\mathbf{R}} \mathbf{C}.$$

If B denotes the Killing form of $\mathfrak{g}$, B is negative-definite on $\mathfrak{k}$, and we let $\mathfrak{p}$ denote the orthogonal complement under B of $\mathfrak{k}$ in $\mathfrak{g}$. Then $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ is a so-called Cartan decomposition of $\mathfrak{g}$, and B is positive-definite on $\mathfrak{p}$.

Let $M = G/K$, the corresponding symmetric space. As $[\mathfrak{k}, \mathfrak{p}] \subset \mathfrak{p}$, B defines an $(\mathrm{Ad}\ K)$-invariant inner product on $\mathfrak{p}$; and since we may identify $\mathfrak{p}$ naturally as the tangent space to M at the identity coset $x_0 = K$, B determines a unique Riemannian metric on M which is invariant under the canonical left G-action.

Assume initially that M is an *irreducible* symmetric space. Then, if one wishes, G can be taken to be a non-compact almost simple group (i.e., $\mathfrak{g}$ is a simple Lie algebra). In that case, the space M admits a homogeneous complex structure, and becomes an *Hermitian* symmetric space, precisely when $\mathfrak{k}$ has a non-trivial center $\mathfrak{z}$. In this case, $\dim \mathfrak{z} = 1$, and $Z = \exp \mathfrak{z}$ is the identity component of the center of K. Let G^{ad} denote the adjoint group of G (i.e., the automorphism group of M) and let K^{ad}, Z^{ad} be the corresponding subgroups of G^{ad}. A choice of $z_0 \in Z^{\mathrm{ad}}$ of order 4 (for which $\mathrm{Ad}\,(z_0^2)$ is a Cartan involution of $\mathfrak{g}$) determines an almost-complex structure on $\mathfrak{p}$: