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*Proof.* By standard diagonalization arguments, there is a language $L$ in $DTIME\,(c_2^n)$ which is not in $DTIME\,(c_1^n)$ for $c_1 < c_2$ [19].

$L$ is then in $ASPACE\,(n)$. Assume Corollary 1 is not true. Then by first transforming $L$ according to the lower bound theorem to the monadic $\forall\exists\exists$ class, and then accepting this language fast, $L$ could be accepted in deterministic time $c_1^n$. □

COROLLARY 2. *For every nondeterministic Turing machine $M$ which accepts the satisfiable formulas of the monadic $\forall\exists$ class, there exists a constant $c$, such that $M$ uses space $cn/\log n$ for infinitely many inputs.*

*Proof.* We use the hierarchy result for $NSPACE$ [35] and the fact that an alternating Turing machine with only one successor configuration for each universal configuration, is a nondeterministic Turing machine. □

## CONCLUSIONS

Alternating Turing machines are a powerful tool in the few areas where applications have been found so far. They can make connections visible, which are not seen otherwise. It seems impossible to find the lower bound for the Ackermann case of the decision problem, without knowing alternating Turing machines. Even knowing the result, a direct description of the computation of a deterministic exponential time bounded Turing machine $M$ by a $\exists^* \forall\exists^*$ formula, without obviously copying the simulation of $M$ by an alternating Turing machine, seems impossible.

We are used to think that nondeterministic machines correspond to existential quantifiers (e.g. satisfiability in propositional calculus), and that alternating machines correspond to a sequence of alternating quantifiers (e.g. quantified boolean formulas, i.e. the theory of $\{0, 1\}$ with equality). This paper shows that this needs not always to be the case.

*Examples*

1. Not only the satisfiability problem of the $\exists^*$ class, but also of the $\forall^*$ class is *NP*-complete (not co-*NP*-complete).
2. Adding an existential quantifier to the $\forall$ prefix class, means moving from a time to a space complexity class.

3. Adding another existential quantifier to the $\forall\exists$ prefix class means moving from a nondeterministic (space) to a deterministic (time) complexity class.

One possible continuation of this work, is to investigate the complexity of the decision problem for formulas with simple quantifier patterns in decidable theories. For most of the decidable theories, huge lower bounds are known, because a class of formulas with so many quantifier alternations, that they hardly appear in practice, is shown to be difficult to decide.

## Acknowledgment

The deterministic lower time bound $c^{n/\log n}$ for the $\exists^*\forall\exists^*$ case has been obtained independently by Harry R. Lewis (Complexity Results for Classes of Quantificational Formulas. *J. of Computer and System Sciences 21*, No. 3, Dec. 1980, pp. 317-353). His method is quite different and uses alternating pushdown automata.

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