

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 27 (1981)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ALTERNATION AND THE ACKERMANN CASE OF THE DECISION PROBLEM
Autor: Fürer, Martin

Bibliographie
DOI: <https://doi.org/10.5169/seals-51744>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 14.03.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

3. Adding another existential quantifier to the $\forall\exists$ prefix class means moving from a nondeterministic (space) to a deterministic (time) complexity class.

One possible continuation of this work, is to investigate the complexity of the decision problem for formulas with simple quantifier patterns in decidable theories. For most of the decidable theories, huge lower bounds are known, because a class of formulas with so many quantifier alternations, that they hardly appear in practice, is shown to be difficult to decide.

ACKNOWLEDGMENT

The deterministic lower time bound $c^{n \setminus \log n}$ for the $\exists^* \forall \exists^*$ case has been obtained independently by Harry R. Lewis (Complexity Results for Classes of Quantificational Formulas. *J. of Computer and System Sciences* 21, No. 3, Dec. 1980, pp. 317-353). His method is quite different and uses alternating pushdown automata.

REFERENCES

- [1] ACKERMANN, Wilhelm. Über die Erfüllbarkeit gewisser Zählausdrücke. *Mathematische Annalen* 100 (1928), pp. 638-649.
- [2] ———. *Solvable Cases of the Decision Problem*. North Holland, Amsterdam, 1954.
- [3] AHO, A. V., J. E. HOPCROFT and J. D. ULLMAN. *The Design and Analysis of Computer Algorithms*. Addison-Wesley, 1974.
- [4] ASSER, G. Das Repräsentantenproblem im Prädikatenkalkül der ersten Stufe mit Identität. *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* 1 (1955), pp. 252-263.
- [5] BENNETT, J. On spectra. Doctoral Dissertation, Princeton University, N.J., 1962.
- [6] BERMAN, L. Precise bounds for Presburger arithmetic and the reals with addition. *Proceedings 18th Symposium on Foundations of Computer Science*, 1977, pp. 95-99.
- [7] BERNAYS, Paul und Moses SCHÖNFINKEL. Zum Entscheidungsproblem der mathematischen Logik. *Mathematische Annalen* 99 (1928), pp. 342-372.
- [8] BÜCHI, J. R. Turing machines and the Entscheidungsproblem. *Mathematische Annalen* 148 (1962), pp. 201-213.
- [9] CHANDRA, A. K., D. C. KOZEN and L. J. STOCKMEYER. Alternation. *Journal of the ACM* 28 (1981), pp. 114-133.
- [10] CHANDRA, A. K. and L. J. STOCKMEYER. Alternation. *Proc. 17th Symposium on Foundations of Computer Science*, 1976, pp. 98-108.
- [11] CHRISTEN, C. Spektren und Klassen elementarer Funktionen. Doctoral Dissertation, ETH Zürich, 1974.
- [12] CHURCH, Alonzo. A note on the Entscheidungsproblem. *Journal of Symbol. Logic* 1 (1936), pp. 40-41; Correction. *ibid.*, pp. 101-102.

- [13] COOK, S. A. Characterization of push-down machines in terms of time-bounded computers. *Journal of the Association for Computing Machinery* 18 (1971), pp. 4-18.
- [14] DREBEN, B. and W. D. GOLDFARB. *The Decision Problem: Solvable Classes of Quantificational Formulas*. Addison-Wesley, Advanced Book Program, Reading, Massachusetts, 1979.
- [15] GAREY, M. R. and D. S. JOHNSON. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman, San Francisco, 1979.
- [16] GÖDEL, Kurt. Ein Spezialfall des Entscheidungsproblems der theoretischen Logik. *Ergebnisse eines mathematischen Kolloquiums* 2, 1932, pp. 27-28.
- [17] ——— Zum Entscheidungsproblem des logischen Funktionenkalküls. *Monatsch. Math. Phys.* 40 (1933), pp. 433-443.
- [18] GOLDFARB, W. D. To appear in: *The Journal of Symbolic Logic*.
- [19] HARTMANIS, J. and R. E. STEARNS. On the computational complexity of algorithms. *Transactions of the American Mathematical Society* 117 (1965), pp. 285-306.
- [20] IBARRA, O. H. Characterization of some tape and time complexity classes of Turing machines in terms of multihead and auxiliary stack automata. *J. Comput. System Sci.* 5 (1971), pp. 88-117.
- [21] JONES, N. G. and A. L. SELMAN. Turing machines and the spectra of first-order formulas. *Journal of Symbolic Logic* 39 (1974), pp. 139-150.
- [22] KAHR, A. S., E. F. MOORE und Hao WANG. Entscheidungsproblem Reduced to the AEA Case. *Proc. Nat. Acad. Sci. USA* 48 (1962), pp. 365-377.
- [23] KALMÁR, László. Über die Erfüllbarkeit derjenigen Zählausdrücke, welche in der Normalform zwei benachbarte Allzeichen enthalten. *Mathematische Annalen* 108 (1933), pp. 466-484.
- [24] KOZEN, D. On parallelism in Turing machines. *Proc. 17th Symp. on Foundations of Computer Science*, 1976, pp. 89-97.
- [25] ——— First order predicate logic without negation is NP-complete. Report No. 77-307, Dept. of Computer Science, Cornell University, Ithaca, NY, 1977.
- [26] LADNER, R. E., R. J. LIPTON and L. J. STOCKMEYER. Alternating Pushdown Automata. *Proc. 19th Symp. on Foundations of Computer Science*, 1978, pp. 92-106.
- [27] LEWIS, Harry R. Complexity of solvable cases of the Decision Problem for Predicate Calculus. *Proc. 19th Symp. on Foundations of Computer Science*, 1978, pp. 35-47.
- [28] ——— *Unsolvable Classes of Quantificational Formulas*. Addison-Wesley, Advanced Book Program, Reading, Massachusetts, 1979.
- [29] LÖWENHEIM, Leopold. Über Möglichkeiten im Relativkalkül. *Mathematische Annalen* 76 (1915), pp. 447-470; English translation in [42], pp. 228-251.
- [30] MOSTOWSKI, A. Concerning a problem of H. Scholz. *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* 2 (1956), pp. 210-214.
- [31] RACKOFF, C. The complexity of theories of the monadic predicate calculus. Technical Report, IRIA, 1975.
- [32] RUZZO, L. Tree-Size Bounded Alternation. *Proceedings of the 11th Annual ACM Symposium on Theory of Computing*, 1979, pp. 352-359.
- [33] SCHÜTTE, Kurt. Untersuchungen zum Entscheidungsproblem der mathematischen Logik. *Mathematische Annalen* 109 (1934), pp. 572-603.
- [34] ——— Über die Erfüllbarkeit einer Klasse von logischen Formeln. *Mathematische Annalen* 110 (1934), pp. 161-194.
- [35] SEIFERAS, J. I., M. J. FISCHER and A. R. MEYER. Refinements of the nondeterministic time and space hierarchies. *Proceedings of 14th IEEE Symposium on Switching and Automata Theory*, 1973, pp. 130-137.
- [36] SCHOENFIELD, J. R. *Mathematical Logic*. Addison-Wesley, Series in Logic, 1967.

- [37] SKOLEM, Thoralf. Logisch-kombinatorische Untersuchungen über die Erfüllbarkeit oder Beweisbarkeit mathematischer Sätze nebst einem Theorem über dichte Mengen. *Videnskapsselskapets skrifter, I. Matematisk-naturvidenskabelig klasse, No. 4* (1920), 36 pp; English translation in [42], pp. 252-263.
- [38] ——— Einige Bemerkungen zur axiomatischen Begründung der Mengenlehre. *Matematikerkongressen i Helsingfors den 4-7 Juli 1922, Den femte skandinaviska matematikerkongressen, Redogörelse* (Akademiska Bokhandeln, Helsinki, 1923), pp. 217-232; English translation in [42], pp. 290-301.
- [39] ——— Über die mathematische Logik. *Norsk matematisk tidsskrift 10* (1928), pp. 125-142; English translation in [42], pp. 508-524.
- [40] SURÁNYI, János. Zur Reduktion des Entscheidungsproblems des logischen Funktionen-kalküls. *Mat. es Fizikai Lapok 50* (1943), pp. 51-74.
- [41] TURING, Alan Mathison. On computable numbers, with an application to the Entscheidungsproblem. *Proc. London Math. Soc. 42* (1936/37), pp. 230-265. A correction, *ibid.* 43 (1937), pp. 544-546.
- [42] VAN HEIJENOORT, Jean. *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931*. Harvard University Press, Cambridge, Massachusetts, 1971.
- [43] WANG, Hao. Proving Theorems by Pattern Recognition II. *The Bell System Technical Journal 40* (1961), pp. 1-41.
- [44] ——— Dominoes and the AEA case of the decision problem. *Proceedings of a Symposium on the Mathematical Theory of Automata*, Polytechnic Institute of Brooklyn, New York, 1962, pp. 23-55.

(Reçu le 28 septembre 1980)

Martin Fürer

Computer Science Department
University of Edinburgh
Edinburgh EH9 3JZ
Scotland