Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 27 (1981)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE RIEMANN-ROCH THEOREM FOR COMPACT RIEMANN SURFACES

Autor: Simha, R. R.

Bibliographie

DOI: https://doi.org/10.5169/seals-51747

we need only show that, if $\sigma \in H^0(X, \mathscr{L})$ is such that $\zeta(\sigma \otimes \gamma) = 0$ for all $\gamma \in H^1(X, K \otimes \mathscr{L}^{-1})$, then $\sigma \equiv 0$. Now choose any $P \in X$, and a coordinate neighbourhood (U, z) of P centred at P such that $\mathscr{L} \mid U \approx \mathcal{O}_U$. Then the covering $\mathfrak{U} = \{U, X - P\}$ is a Leray covering for $\mathscr{L}, K$ and $K \otimes \mathscr{L}^{-1}$ ((3.7)). The $z^n\, dz, n \in \mathbf{Z}$, can all be regarded as elements of $Z^1(\mathfrak{U}, K \otimes \mathscr{L}^{-1})$; let γ_n denote their images in $H^1(X, K \otimes \mathscr{L}^{-1})$. Then clearly $\rho(\sigma \otimes \gamma_n) = 0$ for all n implies that all the coefficients of the Taylor expansion of σ at P with respect to vanish, hence $\sigma \equiv 0$, q.e.d.

(5.9) Serre duality for vector bundles. *For any vector bundle $\mathscr{V}$ on X, let $\mathscr{V}^* = \text{Hom}\, \mathcal{O}_X(\mathscr{V}, \mathcal{O}_X)$. Then the natural pairing*

$$\zeta : H^0(X, \mathscr{V}) \times H^1(X, K \otimes \mathscr{V}^*) \to H^1(X, K) \overset{res}{\tilde{\to}} \mathbf{C}$$

is non-degenerate.

Proof: Arguing as in the proof of (5.8) we see that the map $H^0(X, \mathscr{V}) \to (H^1(X, K \otimes \mathscr{V}^*))^*$ induced by ζ is injective, hence $h^0(X, \mathscr{V}) \leqslant h^1(X, K \otimes \mathscr{V}^*)$. Replacing $\mathscr{V}$ by $K \otimes \mathscr{V}^*$, we also get $h^0(K \otimes \mathscr{V}^*) \leqslant h^1(\mathscr{V})$. But, by induction on rank $\mathscr{V}$, we easily deduce from (5.3) that $\chi(K \otimes \mathscr{V}^*) = -\chi(\mathscr{V})$, hence $h^0(X, \mathscr{V}) = h^1(X, K \otimes \mathscr{V}^*)$. Thus ζ is non-degenerate as before.

REFERENCES

[1] Grauert, H. and R. Remmert. *Theory of Stein Spaces*. Springer-Verlag, 1979.
[2] Gunning, R. C. *Lectures on Riemann surfaces*. Princeton University Press.
[3] Gunning, R. C. and H. Rossi. *Analytic Functions of Several Complex Variables*. Prentice Hall, 1965.
[4] Mumford, D. *Algebraic Geometry I: Complex Projective Varieties*. Springer-Verlag, 1976.
[5] Serre, J.-P. *Groupes Algébriques et Corps de Classes*. Hermann, 1959.

(Reçu le 10 juillet 1980)

R. R. Simha
School of Mathematics
Tata Institute of Fundamental Research
Bombay 400 005
India