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we need only show that, if  $\sigma \in H^0(X, \mathcal{L})$  is such that  $\zeta(\sigma \otimes \gamma) = 0$  for all  $\gamma \in H^1(X, K \otimes \mathcal{L}^{-1})$ , then  $\sigma \equiv 0$ . Now choose any  $P \in X$ , and a coordinate neighbourhood  $(U, z)$  of  $P$  centred at  $P$  such that  $\mathcal{L}|_U \approx \mathcal{O}_U$ . Then the covering  $\mathfrak{U} = \{U, X - P\}$  is a Leray covering for  $\mathcal{L}, K$  and  $K \otimes \mathcal{L}^{-1}$  ((3.7)). The  $z^n dz, n \in \mathbf{Z}$ , can all be regarded as elements of  $Z^1(\mathfrak{U}, K \otimes \mathcal{L}^{-1})$ ; let  $\gamma_n$  denote their images in  $H^1(X, K \otimes \mathcal{L}^{-1})$ . Then clearly  $\rho(\sigma \otimes \gamma_n) = 0$  for all  $n$  implies that all the coefficients of the Taylor expansion of  $\sigma$  at  $P$  with respect to vanish, hence  $\sigma \equiv 0$ , q.e.d.

(5.9) SERRE DUALITY FOR VECTOR BUNDLES. *For any vector bundle  $\mathcal{V}$  on  $X$ , let  $\mathcal{V}^* = \text{Hom } \mathcal{O}_X(\mathcal{V}, \mathcal{O}_X)$ . Then the natural pairing*

$$\zeta : H^0(X, \mathcal{V}) \times H^1(X, K \otimes \mathcal{V}^*) \rightarrow H^1(X, K) \xrightarrow{\text{res}} \mathbf{C}$$

*is non-degenerate.*

*Proof:* Arguing as in the proof of (5.8) we see that the map  $H^0(X, \mathcal{V}) \rightarrow (H^1(X, K \otimes \mathcal{V}^*))^*$  induced by  $\zeta$  is injective, hence  $h^0(X, \mathcal{V}) \leq h^1(X, K \otimes \mathcal{V}^*)$ . Replacing  $\mathcal{V}$  by  $K \otimes \mathcal{V}^*$ , we also get  $h^0(K \otimes \mathcal{V}^*) \leq h^1(\mathcal{V})$ . But, by induction on rank  $\mathcal{V}$ , we easily deduce from (5.3) that  $\chi(K \otimes \mathcal{V}^*) = -\chi(\mathcal{V})$ , hence  $h^0(X, \mathcal{V}) = h^1(X, K \otimes \mathcal{V}^*)$ . Thus  $\zeta$  is non-degenerate as before.

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