

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 28 (1982)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** FROBENIUS RECIPROCITY AND LIE GROUP REPRESENTATIONS ON  $\bar{\Delta}$  COHOMOLOGY SPACES  
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**Bibliographie**  
**DOI:** <https://doi.org/10.5169/seals-52231>

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5. It is possible to formulate Frobenius reciprocity for unitary representations on a Hilbert space  $\mathcal{H}(D)$  of  $L_2$ -solutions of an invariant elliptic differential operator  $D$  on homogeneous bundles over a homogeneous space  $G/H$  whose isotropy subgroup  $H$  is compact modulo the center of  $G$ . Here  $G$  is a connected unimodular Lie group (not necessarily semisimple) subject to some mild structural constraints. In [33] Connes and Moscovici show that  $\mathcal{H}(D)$  decomposes as a finite direct sum of irreducible unitary representations all of which are square-integrable modulo the center of  $G$  and occur with finite multiplicity. They derive for  $\mathcal{H}(D)$  a reciprocity analogous to that expressed for the  $L_2$ -cohomology spaces in Theorem 3.15 and Theorem 4.3.

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(Reçu le 20 janvier 1981)

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