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Artikel: THE REPRESENTATION THEORY OF SL(2, R), A NON-INFINITESIMAL APPROACH

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5. Equivalence of irreducible representations of $SU(1, 1)$ to subrepresentations of the principal series

The first two subsections review some generalities about Gelfand pairs and spherical functions. By using the concepts developed there we can next, in §5.4, translate the problem of classifying the irreducible representations of $SU(1, 1)$ in such a way that the problem can be solved by global methods. For this the generalized Abel transform (§5.3) and the Chebyshev transform pair of Deans (Theorem 5.10) are the main tools. The problem is finally reduced to finding the continuous characters on the convolution algebra $\mathscr{D}_{\text{even}}(\mathbf{R})$ (Prop. 5.7).

5.1. Spherical functions

We remember some of the standard facts about spherical functions (cf. for instance Godement [20], Helgason [25, Ch. X], Faraut [12, Ch. 1]). Let G be a unimodular lcsc. group with compact subgroup K. (G, K) is called a *Gelfand pair* if $C_c(K\backslash G/K)$ is a commutative algebra under convolution. If there is a continuous involutive automorphism α on G such that $\alpha(KxK) = Kx^{-1}K (x \in G)$ then (G, K) is a Gelfand pair. If (G, K) is a Gelfand pair and the irreducible representation τ of G is unitary or K-finite then the representation 1 of K has multiplicity 0 or 1 in τ.

Let (G, K) be a Gelfand pair. A *spherical function* is a function $\phi \neq 0$ on G such that

$$\phi(x)\phi(y) = \int_K \phi(xky)dk, \quad x, y \in G.$$

The nonzero continuous algebra homomorphisms from $C_c(K\backslash G/K)$ (or $C_c^\infty(K\backslash G/K)$ if G is a Lie group) to $\mathbf{C}$ are precisely of the form

$$f \to \int_G f(x)\phi(x^{-1})dx, \tag{5.1}$$

where ϕ is a spherical function. If τ is a K-unitary representation of G and if $\mathscr{H}(\tau)$ contains a K-fixed unit vector v, unique up to a constant factor, then $x \to (\tau(x)v, v)$ is a spherical function.