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Artikel: THE INFLUENCE OF COMPUTERS AND INFORMATICS ON MATHEMATICS AND ITS TEACHING

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We therefore pose the following question:

Question 10. What elements of computer science and informatics should be introduced into the training of teachers, and how can they be prepared and helped to teach mathematics in a new way, consonant with the new computing context? Some experience in this area already exists.

3. The computer as an aid to the teaching of mathematics

3.1. The general effects of computers

The use of computers compels one not only to recognise in the area of experiments a source of mathematical ideas and a field for the illustration of results, but also a place where confrontation will permanently occur between theory and practice. This last poses a problem, which will occur in the training of teachers as well as of students, of promoting the *experimental attitude* (observation, testing, control of variables, ...) alongside, and on a par with, the *mathematical attitude* (conjecture, proof, verification, ...). Does it suffice, to speak, as some people do, of "experimental mathematics"?

We now have a triangle, student-teacher-computer, where previously only a dual relationship existed. Is there not a danger that, in order to preserve as much as possible the traditional student-teacher relationship, students' work on a computer will be restricted to simplistic activities which are "without risk" for the teacher?

Students are bound to be aware (as a result of their environment and the media) of the widespread use of computers as well as their associated peripherals, even interconnecting systems and data banks. They have also seen spectacular graphics displayed on a screen, or traced on a plotter. As a result of this, students have new expectations with respect to teaching in general and that of mathematics in particular. How can the computer be used by and with the students in order to meet these new expectations?

In addition to the changes of interest to which informatics leads, one must also draw attention to the changes in the difficulty of exercises and problems. Not only will the use of a computer change the order of difficulty of exercises, but it will also change the relative difficulties of the various ways of solving the same exercise. How can one arrive at new hierarchies and take them into account when one constructs exercises?

3.2. OBJECTIVES AND MODES OF OPERATION

There are various methods of using a computer in our teaching. The teacher can use the computer like a "blackboard", in the same way as one proceeds when giving demonstrations in the experimental sciences. However, the use of an interactive computer permits a much higher degree of interaction with the audience. This particular mode has been tested in various places, but its wider use depends upon the provision of more, and more varied, software. What are the specific requirements which must be met by such software?

The computer can be used by students, individually or in groups of two or more, in order to accomplish predetermined work (this is really programmed learning adapted to work on the computer: unfortunately, there seems to be little software of this type available having any great mathematical interest). In a similar manner, the computer can provide the student with a permanent and readily accessible form of self-evaluation.

Another use of the computer is for "practical work": the experimental manipulation of mathematical objects in connection with open-ended problems (e.g. statistical treatment of data, geometric explorations, the manipulation of functions, ...).

One sees, then, the need for the development of "software banks" in order both to provide support for teachers and lecturers and also to encourage further improvements. This software, which should be available to all, would be located in "multi-media centres" in the middle of institutions and seen as a means of communication on a par with written texts, films, ...

The preparation of software will call upon the united skills of mathematicians, computer scientists and practising teachers. How should one share out the work in order to produce satisfactory software and within which structural framework?

Finally, another use of the computer, in the school or university environment, is in the context of a "computer club". After an initial period of familiarisation, it is the users/members who are chiefly responsible for determining the paths to follow. This type of work is of relevance not only to students, but also to their teachers. What needs can be identified, therefore, for those who are going to be responsible for the training of teachers?

3.3. The treatment of particular areas

The peripherals used (screen, printer, plotter, ...) determine different ways of using informatics. The adaptation to mathematics poses some general problems, like that of the handling of symbolic writing which is not linear, despite the apparent linear sequence of characters in a normal text. For example, consider the various methods employed to reduce to a linear form mathematical statements often best presented in the form of a tree.

We now consider methods of employing the computer to meet needs to be found in various areas of mathematics which are taught at the levels of education under consideration.

In all of these branches one will note the central rôle of visualisation, of experimentation, of simulation, and of the way in which the computer fosters the generation and refining of conjectures.

First, however, we pose a general question. A certain number of fundamental concepts are used in the teaching of mathematics, often in an implicit manner, for example, intuitive logic, the concepts of a variable, of a function, ... Can informatics help us bring precision to, and increase our understanding of, such concepts?

Statistics and probability; data processing. The computer permits the processing of data on a truly grand scale. Problems of sorting data into classes are no longer relevant. Again, simulation is a tool which can hold a place in probability similar to that of plotting figures in geometry. Thus it is possible, thanks to pseudo-random techniques for selection, to provide "reality" for all types of conceivable situations—betting, decision making, testing ...

Geometry. The production of graphical images (e.g. perspective views of objects in space, orbits) and the concept of computer-aided design (graphics software) are extremely helpful for the development and fostering of intuitions. They make it possible to explore geometric objects and figures and provide access to new figures. What changes does plotting by means of a computer introduce with respect to geometry founded on the use of ruler and compass?

Linear algebra. The algorithmic approach furnishes tools for mathematical demonstrations (e.g. pivotal condensation) and leads us to approach in a different manner the study of such questions as inversion, the solution of systems of equations, and the decomposition of matrices. Moreover, visualisation can give support to intuition, e.g. for the study of eigenvalues and of diagonalisation. Do not such techniques as the simplex method merit a place in our teaching?

Analysis. As a result of the effects of symbolical systems, exercises on differentiation, searching for primitive integrals and finding finite Taylor series, are destined to decrease in importance. On the other hand, the graphical representation of functions and finding approximate solutions of numerical or functional equations will become worthy of additional consideration. Experimentation can provide opportunities for the discovery and formulation of qualitative properties before they are formally proved, for example, for the solution of differential equations. Approximation brings with it problems of convergence, beginning with sequences and series. Moreover, the qualitative aspect of the concept of convergence, the numerical study, leads naturally to the quantitative aspect, speed of convergence. Finally, discretisation provides a further field for experimentation, e.g. for functional equations.

Numbers, numerical analysis. The numbers of a machine are very different from those of a mathematician. This leads one to explore the differences and, *en passant*, to consider the principles of numerical symbolism. In another connection, should we be taking the use of parallel processors for research in numerical analysis into account at the teaching level?

Sets, combinatorics, logic. The methods of working now force one to give operative definitions (the enumeration of surjections $S(n, p)$ is a simple example of recursivity, which also allows one to give a working meaning to a surjection). In this area, too, the rapid production of numerical results permits easy exploration and the devising of conjectures. Does the learning of formulae by experience constitute a particular current interest in this field?

In traditional fields of study there are subjects which demand new and special attention because of the particular characteristics of working on a machine: that it uses discrete methods. It is important, therefore, to pay attention to theoretical approaches to discrete topics (e.g. difference equations); at this time, complete courses of discrete mathematics are being proposed for students. Is it really true that this provides us with a new theme for teaching?

3.4. Assessment and recording

The teacher often understands the assessment of his pupils' learning in the restricted sense of evaluation through examinations, while assessment of teaching is usually ignored. The computer, however, now makes possible a variety of ways of controlling assessment, ranging from the presentation of exercises to students to the management of individual files. The use of the computer to construct and to conduct evaluatory tests has hardly been experimented with up to now except in the teaching of computer science itself. Should we foresee a general development in the growth of examinations "on a computer", and if so how are such tests to be designed?

The notion of control and evaluation can also be extended to what happens when we use a computer. The juxtaposition of the output from a computer with mathematical results is specially relevant to such "experimental control". At the end of this report it is time to mention the usefulness of results which do not correspond to what has been foreseen and to those programs which do not function perfectly. It is obviously helpful to recall that frequently programs will not work at the first attempt. What is the mathematical interest in such mistakes?

3.5. The training of teachers

We have referred above to the problem of the content of teacher-training. It is equally advisable to question the form that this training should take, particularly the provision of in-service education for practising teachers. What can be envisaged if we think of "light" in-service training —day-release or short-term courses—and what if teachers can be given at least one year's complete leave of absence from teaching? But even this last is not sufficient considered in the context of the gradual evolution of materials and software. Here it would seem essential to open local support centres designed to provide a follow-up to such courses, to supply up-to-date software and to encourage teaching experiments. It would be a great pity if interest in computers and informatics resulted in the establishment of "heavy" administrative machinery, distant from most teachers, in which decisions relating to teaching were taken. What networks (local, regional, national, international) is it advisable, therefore, to set up and what type of connections must be established between them?
