

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 32 (1986)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON CONSECUTIVE VALUES OF THE LIOUVILLE FUNCTION
Autor: Hildebrand, Adolf
Kurzfassung
DOI: <https://doi.org/10.5169/seals-55088>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 15.03.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

ON CONSECUTIVE VALUES OF THE LIOUVILLE FUNCTION

by Adolf HILDEBRAND

ABSTRACT: It is shown that for every choice of $\varepsilon_i = \pm 1, i = 1, 2, 3$, there exist infinitely many positive integers n , such that $\lambda(n+i) = \varepsilon_i, i = 1, 2, 3$, where λ denotes the Liouville function. ¹⁾

1. INTRODUCTION

Let $\lambda(n)$ denote the Liouville function, i.e. $\lambda(n) = +1$ or -1 according as the number of prime factors of n (counted with multiplicity) is even or odd. It is natural to expect that the sequence $(\lambda(n))$ behaves like a random sequence of \pm signs. A particularly attractive and highly plausible conjecture is that every finite "block" of \pm signs occurs in this sequence infinitely often, i.e. for any given numbers $\varepsilon_i = \pm 1, 1 \leq i \leq k$, there are infinitely many integers $n \geq 1$, such that

$$\lambda(n+i) = \varepsilon_i \quad (1 \leq i \leq k).$$

Whereas for $k = 1$ and $k = 2$ this conjecture holds trivially, there are no results known in the literature for larger values of k . In [1, p. 95, problem 56], Chowla states the above conjecture and remarks that "for $k \geq 3$, this seems an extremely hard conjecture". The purpose of this paper is to prove the conjecture in the first non-trivial case $k = 3$.

THEOREM. *For any choice of $\varepsilon_i = \pm 1, i = 1, 2, 3$, there are infinitely many positive integers n such that*

$$(1) \quad \lambda(n+i) = \varepsilon_i \quad (i=1, 2, 3).$$

We shall use for the proof an "ad hoc" method, which leads in a relatively simple way and using only very elementary arguments to the

¹⁾ 1980 A.M.S. Subject Classification: Primary 10 H 25, Secondary 10 K 20, 10 A 20.