Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 32 (1986)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON CONSECUTIVE VALUES OF THE LIOUVILLE FUNCTION

Autor: Hildebrand, Adolf

Kurzfassung

DOI: https://doi.org/10.5169/seals-55088

L'Enseignement Mathématique, t. 32 (1986), p. 219-226

ON CONSECUTIVE VALUES OF THE LIOUVILLE FUNCTION

by Adolf HILDEBRAND

ABSTRACT: It is shown that for every choice of $\varepsilon_i = \pm 1, i = 1, 2, 3,$ there exist infinitely many positive integers n, such that $\lambda(n+i) = \varepsilon_i$, $i = 1, 2, 3,$ where λ denotes the Liouville function. [1]

1. INTRODUCTION

Let $\lambda(n)$ denote the Liouville function, i.e. $\lambda(n) = +1$ or -1 according as the number of prime factors of n (counted with multiplicity) is even or odd. It is natural to expect that the sequence $(\lambda(n))$ behaves like a random sequence of $\pm$ signs. A particularly attractive and highly plausible conjecture is that every finite "block" of $\pm$ signs occurs in this sequence infinitely often, i.e. for any given numbers $\varepsilon_i = \pm 1, 1 \leqslant i \leqslant k$, there are infinitely many integers $n \geqslant 1$, such that

$$\lambda(n+i) = \varepsilon_i \quad (1 \leqslant i \leqslant k).$$

Whereas for $k = 1$ and $k = 2$ this conjecture holds trivially, there are no results known in the literature for larger values of k. In [1, p. 95, problem 56], Chowla states the above conjecture and remarks that "for $k \geqslant 3$, this seems an extremely hard conjecture". The purpose of this paper is to prove the conjecture in the first non-trivial case $k = 3$.

THEOREM. *For any choice of* $\varepsilon_i = \pm 1, i = 1, 2, 3,$ *there are infinitely many positive integers* n *such that*

$$\lambda(n+i) = \varepsilon_i \quad (i = 1, 2, 3). \tag{1}$$

We shall use for the proof an "ad hoc" method, which leads in a relatively simple way and using only very elementary arguments to the

[1]) *1980 A.M.S. Subject Classification:* Primary 10 H 25, Secondary 10 K 20, 10 A 20.