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### § 3. THE ALGEBRAIC PROPERTIES OF $\mathcal{E}$ (See [SKK], [Bj])

3.1. In the preceding section, we introduced the notion of micro-differential operators. The ring  $\mathcal{E}$  of micro-differential operators has nice algebraic properties similar to those of the ring of holomorphic functions.

Let us recall some definitions of finiteness properties.

*Definition 3.1.1.* Let  $\mathcal{A}$  be a sheaf of rings on a topological space  $S$ .

- (1) An  $\mathcal{A}$ -module  $\mathcal{M}$  is called *of finite type* (resp. *of finite presentation*) if for any point  $x \in X$  there exists a neighborhood  $U$  and an exact sequence  $0 \leftarrow \mathcal{M}|_U \leftarrow \mathcal{A}^p|_U$  (resp.  $0 \leftarrow \mathcal{M}|_U \leftarrow \mathcal{A}^p|_U \leftarrow \mathcal{A}^q|_U$ ).
- (2)  $\mathcal{M}$  is called *pseudo-coherent*, if any submodule of finite type defined on an open subset is of finite presentation. If  $\mathcal{M}$  is pseudo-coherent and of finite type, then  $\mathcal{M}$  is called *coherent*.
- (3)  $\mathcal{M}$  is called *Noetherian* if  $\mathcal{M}$  satisfies the following properties:
  - (a)  $\mathcal{M}$  is coherent.
  - (b) For any  $x \in X$ ,  $\mathcal{M}_x$  is a Noetherian  $\mathcal{A}_x$ -module (i.e. any increasing sequence of  $\mathcal{A}_x$ -submodules is stationary).
  - (c) For any open subset  $U$ , any increasing sequence of coherent  $(\mathcal{A}|_U)$ -submodules of  $\mathcal{M}|_U$  is locally stationary.

As for the sheaf of holomorphic functions, we have

**THEOREM 3.1.1** ([SKK] Chap. II, Thm. 3.4.1, Prop. 3.2.7). *Let  $\overset{\circ}{T}^*X$  denote the complement of the zero section in  $T^*X$ .*

- (1)  $\mathcal{E}_X$  and  $\mathcal{E}_X(0)$  are Noetherian rings on  $T^*X$ .
- (2)  $\mathcal{E}_X$  is flat over  $\pi^{-1}\mathcal{D}_X$ .
- (3)  $\mathcal{E}_X(\lambda)|_{\overset{\circ}{T}^*X}$  is a Noetherian  $\mathcal{E}_X(0)|_{\overset{\circ}{T}^*X}$ -module.
- (4) For  $p \in T^*X$ ,  $\mathcal{E}_X(0)_p$  is a local ring with the residual field  $\mathbb{C}$ .
- (5) A coherent  $\mathcal{E}_X$ -module is pseudo-coherent over  $\mathcal{E}_X(0)$ .

### § 4. VARIANTS OF $\mathcal{E}$ (See [SKK], [Bj], [S])

4.1. We have defined the sheaf of rings  $\mathcal{E}$ . However we can introduce other sheaves of rings, similar to  $\mathcal{E}$ , which makes the theory transparent.