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§ 3. The Algebraic Properties of $\mathscr{E}$ (See [SKK], [Bj])

3.1. In the preceding section, we introduced the notion of micro-differential operators. The ring $\mathscr{E}$ of micro-differential operators has nice algebraic properties similar to those of the ring of holomorphic functions.

Let us recall some definitions of finiteness properties.

Definition 3.1.1. Let $\mathscr{A}$ be a sheaf of rings on a topological space S.

(1) An $\mathscr{A}$-module $\mathscr{M}$ is called *of finite type* (resp. *of finite presentation*) if for any point $x \in X$ there exists a neighborhood U and an exact sequence $0 \leftarrow \mathscr{M}|_U \leftarrow \mathscr{A}^p|_U$ (resp. $0 \leftarrow \mathscr{M}|_U \leftarrow \mathscr{A}^p|_U \leftarrow \mathscr{A}^q|_U$).

(2) $\mathscr{M}$ is called *pseudo-coherent*, if any submodule of finite type defined on an open subset is of finite presentation. If $\mathscr{M}$ is pseudo-coherent and of finite type, then $\mathscr{M}$ is called *coherent*.

(3) $\mathscr{M}$ is called *Noetherian* if $\mathscr{M}$ satisfies the following properties:

(a) $\mathscr{M}$ is coherent.

(b) For any $x \in X$, $\mathscr{M}_x$ is a Noetherian $\mathscr{A}_x$-module (i.e. any increasing sequence of $\mathscr{A}_x$-submodules is stationary).

(c) For any open subset U, any increasing sequence of coherent $(\mathscr{A}|_U)$-submodules of $\mathscr{M}|_U$ is locally stationary.

As for the sheaf of holomorphic functions, we have

THEOREM 3.1.1 ([SKK] Chap. II, Thm. 3.4.1, Prop. 3.2.7). *Let* $\mathring{T}^*X$ *denote the complement of the zero section in* T^*X.

(1) $\mathscr{E}_X$ *and* $\mathscr{E}_X(0)$ *are Noetherian rings on* T^*X.

(2) $\mathscr{E}_X$ *is flat over* $\pi^{-1}\mathscr{D}_X$.

(3) $\mathscr{E}_X(\lambda)|_{\mathring{T}^*X}$ *is a Noetherian* $\mathscr{E}_X(0)|_{\mathring{T}^*X}$*-module.*

(4) *For* $p \in T^*X$, $\mathscr{E}_X(0)_p$ *is a local ring with the residual field* **C**.

(5) *A coherent* $\mathscr{E}_X$*-module is pseudo-coherent over* $\mathscr{E}_X(0)$.

§ 4. Variants of $\mathscr{E}$ (See [SKK], [Bj], [S])

4.1. We have defined the sheaf of rings $\mathscr{E}$. However we can introduce other sheaves of rings, similar to $\mathscr{E}$, which makes the theory transparent.