

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 32 (1986)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: TREES, TAIL WAGGING AND GROUP PRESENTATIONS
Autor: Armstrong, M. A.
Kapitel: 3. Tail wagging
DOI: <https://doi.org/10.5169/seals-55090>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 15.03.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

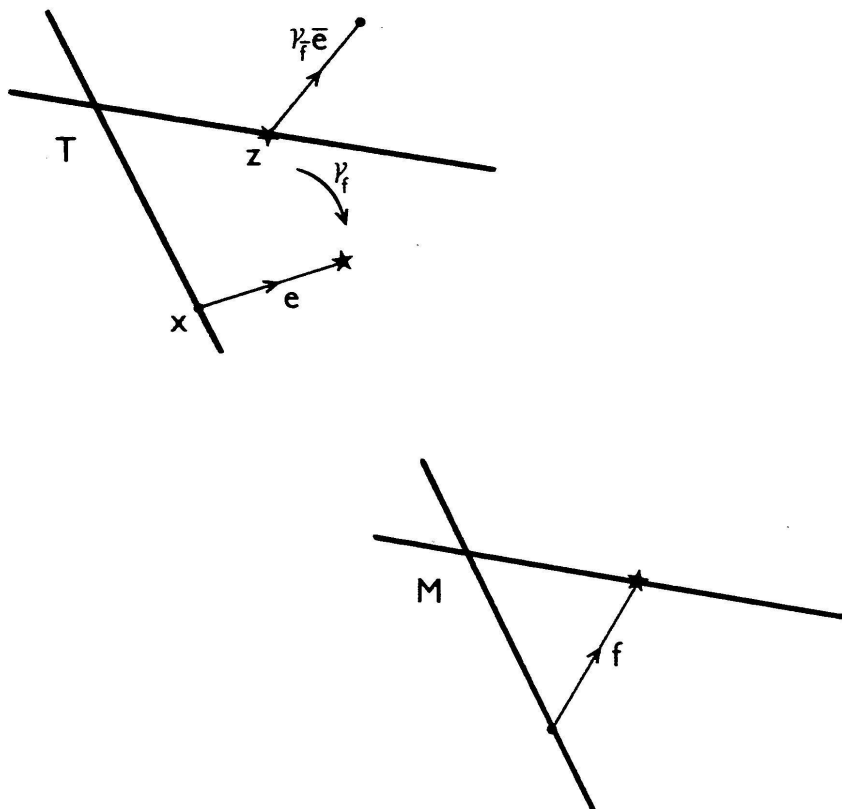


FIGURE 1

3. TAIL WAGGING

With the notation established above let $*G_w$ denote the free product of the stabilizers of the vertices of T , and F the free group generated by symbols λ_f , one for each edge f of X/G . Let R be the normal consequence in $(*G_w)*F$ of the words

$$\begin{aligned} \lambda_f & \quad (f \text{ an edge of } M), \\ \lambda_{\bar{f}} \lambda_f & \quad \text{and} \\ \lambda_{\bar{f}} g_x \lambda_f (\gamma_{\bar{f}} g \gamma_f)_z^{-1} \end{aligned}$$

We shall produce an isomorphism

$$\psi: G \rightarrow [(*G_w)*F]/R.$$

Choose a vertex v of T as base point. If $g \in G$ fixes v set

$$\psi(g) = g_v R$$

where as usual g_v is the element g interpreted as a member of G_v . If g moves v then it sends it outside T because no two vertices of T lie in the same orbit. Let $e_1 e_2 \dots e_n$ be the geodesic which joins v to gv and suppose e_m is the first edge that is *not* in T . The path $e_m e_{m+1} \dots e_n$ will be called the *tail* of $\overrightarrow{v gv}$. Let x_1 be the initial vertex of e_m . Project e_m into X/G to give an edge f_1 . The canonical lift e^1 of f_1 into X has its initial vertex in T , so $i(e^1) = x_1$. Choose an element $a_{x_1} \in G_{x_1}$ which sends e^1 to e_m . Let

$$e_k^1 = (\gamma_{f_1} a_{x_1}^{-1}) e_k$$

for $m+1 \leq k \leq n$, and replace $e_1 e_2 \dots e_n$ by the new path $e_{m+1}^1 e_{m+2}^1 \dots e_n^1$. We call this process *tail wagging*. Our new path begins at

$$z_1 = t(\gamma_{f_1} e^1) = i(e_{m+1}^1)$$

which is a vertex of T and ends at $(\gamma_{f_1} a_{x_1}^{-1} g)v$, see Figure 2. We walk along it to the first point x_2 where it quits T and repeat the above

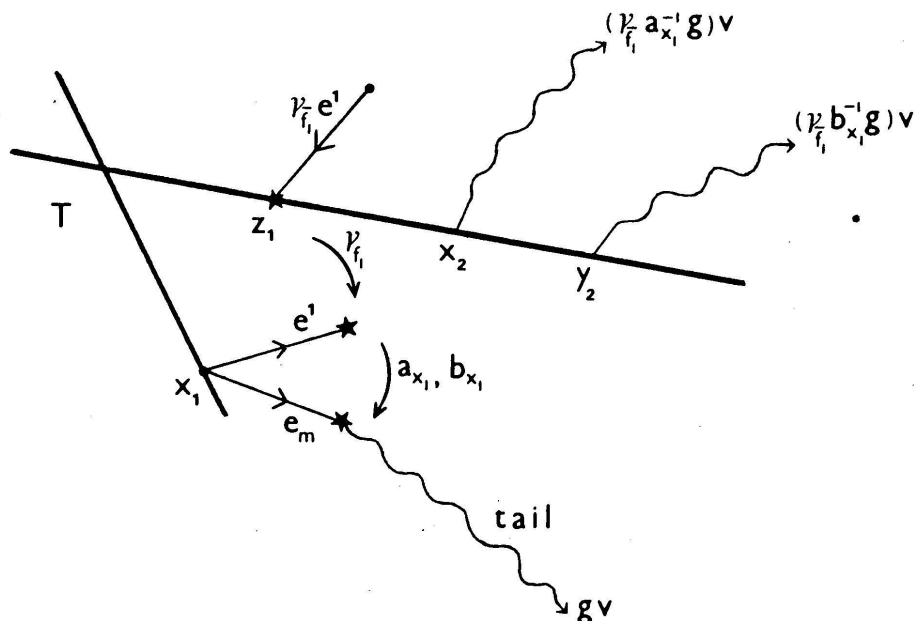


FIGURE 2

procedure. Since we shorten the tail at each step we eventually obtain a path which lies entirely in T and ends at say

$$(\gamma_{\bar{f}_r} a_{x_r}^{-1} \dots \gamma_{\bar{f}_2} a_{x_2}^{-1} \gamma_{\bar{f}_1} a_{x_1}^{-1} g)v.$$

Then $\gamma_{\bar{f}_r} a_{x_r}^{-1} \dots \gamma_{\bar{f}_1} a_{x_1}^{-1} g$ must fix v , say $\gamma_{\bar{f}_r} a_{x_r}^{-1} \dots \gamma_{\bar{f}_1} a_{x_1}^{-1} g = a_v \in G_v$.

We now have

$$g = a_{x_1} \gamma_{f_1} \dots a_{x_r} \gamma_{f_r} a_v$$

and we somewhat optimistically define

$$\psi(g) = a_{x_1} \lambda_{f_1} \dots a_{x_r} \lambda_{f_r} a_v R.$$

4. AN INEFFICIENT CHOICE

Is ψ well defined? The geodesic from v to gv is certainly unique, as is the first point x_1 where it leaves T and its first edge e_m outside T . Both the edge e^1 and the group element γ_{f_1} are now determined by our original construction. The only ambiguity at this stage is the choice of the element $a_{x_1} \in G_{x_1}$ which maps e^1 to e_m . A different choice b_{x_1} will give a path from z_1 to $(\gamma_{\bar{f}_1} b_{x_1}^{-1} g)v$ which leaves T for the first time at say y_2 . The first edge outside T will project to an edge f'_2 of X/G and so on until eventually we have g expressed as

$$g = b_{x_1} \gamma_{f_1} b_{y_2} \gamma_{f'_2} \dots b_{y_s} \gamma_{f'_s} b_v.$$

We must show that $a_{x_1} \lambda_{f_1} a_{x_2} \lambda_{f_2} \dots a_{x_r} \lambda_{f_r} a_v$ and $b_{x_1} \lambda_{f_1} b_{y_2} \lambda_{f'_2} \dots b_{y_s} \lambda_{f'_s} b_v$ determine the same left coset of R in $(*G_w)*F$.

Agree to select a_{x_1} from G_{x_1} so that the tail of the resulting path is as long as possible. Continue in this way selecting $a_{x_2}, a_{x_3} \dots$ so as to maximise the length of the tail at each stage. We shall compare any other set of choices with this rather inefficient selection.

Both a_{x_1} and b_{x_1} map e^1 to e_m , so $c = a_{x_1}^{-1} b_{x_1}$ must fix e^1 . Also, due to our particular selection of a_{x_1} , the geodesic from z_1 to x_2 is left fixed by $\gamma_{\bar{f}_1} c \gamma_{f_1}$. Therefore