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## 5. LINEARIZATION

5.1. The groups  $E_s^U$  can be viewed, through the homomorphism  $\phi: E_s^U \rightarrow \pi_s(U)$  in 3.1, as “linear homotopy groups” of  $U$ . This means that we consider maps of  $S^s$  into  $U$  via some  $U(n)$  which are linear in the coordinates  $x_0, x_1, \dots, x_s$  of  $\mathbf{R}^{s+1} \supset S^s$ ; and linear nullhomotopies, i.e., extensions to  $S^{s+1} \rightarrow U(n)$  linear in  $x_0, x_1, \dots, x_{s+1}$ . It is an immediate corollary of Theorem B that these linear homotopy groups  $\pi_s^{\text{lin}}(U)$  are isomorphic to the  $\pi_s(U)$  by the obvious imbedding  $\pi_s^{\text{lin}}(U) \rightarrow \pi_s(U)$ . In other words:

Any map  $S^s \rightarrow U$  is homotopic to a linear map, and if a linear map  $S^s \rightarrow U$  is nullhomotopic then it admits a linear nullhomotopy.

Similar statements hold, of course, for  $\pi_s(O)$  and  $\pi_s(Sp)$ .

5.2. If these linearization phenomena could be established directly (by some approximation procedure) one would obtain a very transparent proof of the Bott periodicity theorems for  $\pi_s(U)$ ,  $\pi_s(O)$ , and  $\pi_s(Sp)$ , in the sense that they would be reduced to the algebraic computation of  $E_s^U$ ,  $E_s^O$ , and  $E_s^{Sp}$  as carried out here.

5.3. Linear maps  $S^s \rightarrow U$  via  $U(n)$ , etc., are given explicitly in terms of HR-matrices; thus the coefficients involve  $0, \pm 1, \pm i$  only. Such maps have a meaning over very general fields instead of  $\mathbf{R}$  and  $\mathbf{C}$ , and one should compare the corresponding linear homotopy groups with homotopy groups defined by means of algebraic maps.

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