Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 35 (1989)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: HURWITZ-RADON MATRICES AND PERIODICITY MODULO 8
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Bibliographie
DOI: https://doi.org/10.5169/seals-57365

5. LINEARIZATION

5.1. The groups E_s^U can be viewed, through the homomorphism $\phi: E_s^U \to \pi_s(U)$ in 3.1, as "linear homotopy groups" of U. This means that we consider maps of S^s into U via some $U(n)$ which are linear in the coordinates $x_0, x_1, \dots, x_s$ of $\mathbf{R}^{s+1} \supset S^s$; and linear nullhomotopies, i.e., extensions to $S^{s+1} \to U(n)$ linear in $x_0, x_1, \dots, x_{s+1}$. It is an immediate corollary of Theorem B that these linear homotopy groups $\pi_s^{\text{lin}}(U)$ are isomorphic to the $\pi_s(U)$ by the obvious imbedding $\pi_s^{\text{lin}}(U) \to \pi_s(U)$. In other words:

Any map $S^s \to U$ is homotopic to a linear map, and if a linear map $S^s \to U$ is nullhomotopic then it admits a linear nullhomotopy.

Similar statements hold, of course, for $\pi_s(O)$ and $\pi_s(Sp)$.

5.2.If these linearization phenomena could be established directly (by some approximation procedure) one would obtain a very transparent proof of the Bott periodicity theorems for $\pi_s(U)$, $\pi_s(O)$, and $\pi_s(Sp)$, in the sense that they would be reduced to the algebraic computation of E_s^U, E_s^O, and E_s^{Sp} as carried out here.

5.3. Linear maps $S^s \to U$ via $U(n)$, etc., are given explicitly in terms of HR-matrices; thus the coefficients involve 0, ± 1, $\pm i$ only. Such maps have a meaning over very general fields instead of $\mathbf{R}$ and $\mathbf{C}$, and one should compare the corresponding linear homotopy groups with homotopy groups defined by means of algebraic maps.

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(Reçu le 15 décembre 1988)

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