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STATE MODELS FOR LINK POLYNOMIALS

by Louis H. KAUFFMAN

I. INTRODUCTION

This paper gives state models for the oriented (Homfly) ([32], [39], [68]) and semi-oriented (Kauffman) ([48], [49], [43]) polynomials.

The models are state models in the most general spirit of that term. That is, they are summations over diagrammatic states. Each state is obtained from the given link diagram by processes of splicing and labelling. Each state contributes to the summation a product of vertex weights together with a global evaluation. Such a description is parallel to the form of a vertex model or partition function in statistical mechanics [10].

In fact, a number of already known states models are tightly interrelated with statistical mechanics. The author's bracket model for the original Jones polynomial is a knot diagrammatic version of the Potts model (via an ice-model translation) ([44], [51]). The author's states model for the Conway polynomial [42] can be seen as the low temperature limit of a generalized Potts model [57]. Jones and Turaev ([40], [93]) have given models of specializations of both the Homfly and Kauffman polynomials that are also in the form of partition functions (called here Yang-Baxter models, see section 7 of this paper. See also [25], [58]).

One might hope that a state model would make the verification of the existence and properties of a polynomial easy — via the directness of the definition. This is certainly the case for the bracket. And it is also true for the FKT [42] model for the Conway polynomial (modulo a combinatorial theorem, and some algebra). In the case of the Yang-Baxter models, the work is concentrated in seeing that the associated Yang-Baxter "tensor" satisfies certain properties (Yang-Baxter equation and an inversion relation). This part can be clarified using the geometry of link diagrams ([55], [58]).

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The models given in this paper do not share this ease of definition. Their well-definedness is just as easy or difficult as the standard inductive proof of the existence of the polynomial in question. To discriminate them from the Yang-Baxter and other models, I shall call them *skein models.* The skein models are, in fact, seen as translations (into the language of state models) of the recursive process of calculation due to John H. Conway ([16]). The term *skein* is due to Conway. It refers to all the knots and links associated with a given link that are obtained via splicing or switching some of its crossings. Skein calculation uses the knots and links in this skein.

The first skein model was discovered by François Jaeger [33], via a matrix inversion technique. In this paper I show that Jaeger's idea fits into the more general scheme of skein calculation, and hence applies fully to both of the known two-variable skein polynomials, and to their graph embedding generalizations ([56], [74]).

As explained herein, the skein models appear as tautologous — particularly to anyone who has written a computer program to calculate knot polynomials. This is the virtue of our approach! What is significant is that we have, in fact, been in possession of general states models for these polynomials for years. It took Jaeger's observation about standardized basepoints (called here a *template* — see section 2) to show what we already knew.

It is useful to know that general states models exist. And it is very fascinating to compare the form of the general models to the forms of specific models previously known (bracket, FKT, Yang-Baxter). See sections 7, 8, 9 of this paper, particularly section 9 for a discussion of possible physical interpretations. The appendix is a discussion of state model formalism.

II. Skein polynomials

This paper concentrates on two polynomials — the *Homfly* ([24]) or *oriented skein polynomial* and the *Kauffman* or *semi-oriented skein polynomial* ([43], [49]). In both cases it is convenient to first define a polynomial that is an invariant of regular isotopy ([49]), and then normalize the regular isotopy invariant to obtain the corresponding skein polynomial.