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PROPOSITION 1. *If E is a linearly ordered set with m elements, where m is an arbitrary positive integer, then the permutation of E which transforms the elements to reverse order has signature $(-1)^{\binom{m}{2}}$.*

PROPOSITION 2. *If E and F are disjoint linearly ordered sets with m and n elements respectively, where m and n are arbitrary positive integers, then the permutation of $E \cup F$ which transforms the elements from the order in which all elements of E precede all elements of F to the order in which all elements of F precede all elements of E has signature $(-1)^{mn}$.*

PROPOSITION 3. *If E and F are linearly ordered sets with m and n elements respectively, where m and n are arbitrary positive integers, then the permutation of $E \times F$ which transforms the elements from lexicographic (row) order to dual-lexicographic (column) order has signature $(-1)^{\binom{m}{2}\binom{n}{2}}$.*

The simplest method of proof in each case is to count the number of inversions. From the foregoing we see that Proposition 1 is substantially the first supplementary law, Proposition 2 plays a certain auxiliary role in regard to the definition of the Jacobi symbol, and Proposition 3 may be viewed as comprising the combinatorial kernel of the reciprocity law.

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