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- (iii) $(h_{m,1})^{-1}(T) \cap C_m(S)$ is infinite,
- (iv) There exists a finite covering $S_{m,n}$ of S such that the fiber product of $h_{m,n}$ with $S_{m,n}$ is Galois, Abelian and of positive degree.

Let J denote the Jacobian scheme of C over S . Let $a: C \rightarrow J$ be an Albanese morphism. Let p be a prime. Let \bar{T} denote the closure of $a(T)$ in $J(S) \otimes \mathbf{Z}_p$. Since $a(T)$ is infinite it follows from the Mordell-Weil Theorem that there exists a $t \in \bar{T} - a(T)$. Let $t_n \in T$ such that $t - a(t_n) \in p^n J(S)$. Let C_n denote the normalization of the fiber-product of C and J via the map $H_n: x \rightarrow p^n x + t_n$ and $h_{n,1}$ the natural map from C_n to C . It follows that C_n is defined over S and since $H_m(J(S)) \supseteq \{t_n: m \mid n\}$ that $h_{n,1}(C_m(S))$ contains an infinite subset of T .

All that remains is to exhibit the maps $h_{m,n}$. Clearly, $t_m - t_n = p^n r_{m,n}$ for some $r_{m,n} \in J(S)$. Let $H_{m,n}$ denote the map $x: p^{m-n}x + r_{m,n}$. Then $H_{m,k} = H_{n,k} \circ H_{m,n}$. It follows that $H_{m,n}$ pulls back to a morphism $h_{m,n}: C_m \rightarrow C_n$. It is easy to see that this morphism becomes Abelian after adjoining the p^{m-n} -torsion points on J . This proves the proposition. \square

Remark. One can also prove the above proposition with the condition $n \leq m$ replaced by $n \mid m$.

3. COROLLARIES OF THE THEOREM OF THE KERNEL

LEMMA 3.3.1. *Suppose $g: X' \rightarrow X$ is a morphism of smooth proper schemes with geometrically connected fibers over S . Then if $\mu \in PF(X'/S)$ and $s, t \in X(S)$, $(g^*\mu)(s, t) = \mu(g \circ s, g \circ t)$.*

Proof. This follows easily from Lemma 1.3.2. \square

Suppose J is the Jacobian of C over S and g is an Albanese morphism, then since $g^*: H_{DR}^1(J/S) \rightarrow H_{DR}^1(C/S)$ is an isomorphism $g^*: PF(J/S) \rightarrow PF(C/S)$ is an isomorphism.

LEMMA 3.3.2. *Let μ be a fixed Picard-Fuchs differential equation on C/S . Then $\{\mu(s, t): s, t \in C(S)\}$ lies in a finite dimensional subspace of $K[S]$ over K .*

Proof. Suppose $\tilde{\mu} \in PF(J/S)$ such that $g^*\tilde{\mu} = \mu$. The lemma follows from the Mordell-Weil theorem which together with the Theorem of the kernel implies that $J(S)$ modulo the kernel of the homomorphism $s \rightarrow \tilde{\mu}(e, s)$ is a finitely generated Abelian group. \square

LEMMA 3.3.3. *Suppose A is an Abelian scheme over S such that $[W_{A/S}] = H_{DR}^1(A/S)$ and $g: C \rightarrow A$ is a non-constant morphism over S . Fix $s \in C(S)$. Then the set $T = \{t \in C(S) : (g^*\mu)(s, t) = 0 \text{ for all } \mu \in PF(A/S)\}$ is of bounded height.*

Proof. Let A' denote the smallest Abelian subscheme of A over S containing $g(C)$. Since the map $g^*: PF(A/S) \rightarrow PF(A'/S)$ is surjective and $[W_{A/S}] = H_{DR}^1(A/S)$, it follows from Proposition 2.1.2 that $g(T)$ is contained in a translation of the group of constant sections of A'/S . Hence, $g(T)$ is a set of bounded height. Finally, since $C \rightarrow g(C)$ is a finite morphism, it follows that T is a set of bounded height. \square

In particular,

COROLLARY 3.3.4. *Suppose A is an Abelian scheme over S such that $\kappa_{A/S}$ is an isomorphism and $g: C \rightarrow A$ is a non-constant morphism over S . Fix $s \in C(S)$. Then the set $\{t \in C(S) : (g^*\mu_\omega)(s, t) = 0 \text{ for all } \omega \in \omega_{A/S}\}$ is of bounded height.*

4. PROOF OF MORDELL'S CONJECTURE

PROPOSITION 3.4.1. *Suppose the kernel of the $\kappa_{C/S}$ has rank at least 2 over $K[S]$, then the points of $C(S)$ have bounded height.*

Proof. Suppose $C(S)$ contains points of arbitrarily large height. Fix $s \in C(S)$. By shrinking S , if necessary, we may suppose that there exists a function $z \in K[S]$ such that $\Omega_S^1 = K[S]dz$ and there exists a finite covering \mathcal{L} of C by affine opens U and functions $v_U \in \mathcal{O}_C(U)$ such that $s \in U(S)$, and $\Omega_C^1(U)$ is spanned by dz and dv_U . We may also suppose that $s^*v_U = 0$ by replacing v_U with $v_U - (s \circ f)^*v_U$ if necessary. For $U \in \mathcal{L}$, $u \in \mathcal{O}_C(U)$ we define $\partial_{U,z}u$ and $\partial_{U,v}u$ by the equation

$$du = \partial_{U,z}udz + \partial_{U,v}udv_U.$$

Then $\partial_{U,z}$ is a lifting of $\partial = : \partial/\partial z$. We set $\mu(t) = \mu(s, t)$ for

$$\mu \in PF = : PF(C/S)$$

and $t \in C(S)$.

Let ω_1 and ω_2 be two independent elements in the kernel of $\kappa_{C/S}$. It follows that there exist ω'_1 and $\omega'_2 \in \omega_{C/S}$ such that

$$\partial[\omega'_i] = [\omega'_i].$$