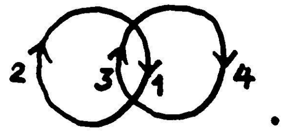
Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 36 (1990)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: STATE MODELS FOR LINK POLYNOMIALS

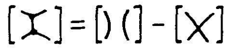
Autor: Kauffman, Louis H.

Bibliographie

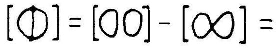
DOI: https://doi.org/10.5169/seals-57900



There are many instances of this sort of expansion outside of the theory of knots and links. For example, the following expansion (compare [78]) for *trivalent plane graphs G*



gives states that are locally four-valent plane graphs. If the value of a state S is taken to be *three raised to the number of crossing circuits in* S, then $[G]$ is the number of colorings of the edges of G with three colors so that three distinct colors meet at each vertex of G. The existence of such a coloring for a trivalent plane graph is well known to be equivalent to finding a four-coloring of its faces so that no two faces that share an edge receive the same color. It is a delicate matter to determine when $[G]$ is non-zero.

 $= 3^2 - 3 = 6$.

Other conventions, more closely related to tensor formalisms are discussed in [78] and [58].

In general, these pictorial expansions are a way to express the vertex weights of a model in a fashion that is easy to relate with the geometry of the diagrams themselves.

REFERENCES

[1] Akutsu, Y. and W. Wadati. Knot invariants and critical statistical systems. *J. Phys. Soc. Japan 56* (1987), 839-842.

[2] Akutsu, Y. and W. Wadati. Exactly solvable models and new link polynomials. I. *N*-state vertex models. *J. Phys. Soc. Japan 56* (1987), 3039-3051.

[3] Akutsu, Y., T. Deguchi and W. Wadati. Exactly solvable models and new link polynomials II. Link polynomials for closed 3-braids. *J. Phys. Soc. Japan 56* (1987), 3464-3479.

[4] Akutsu, Y., T. Deguchi and M. Wadati. Exactly solvable models and new link polynomials III. Two-variable polynomial invariants. (To appear).

[5] Akutsu, Y. and M. Wadati. Knots, links, braids and exactly solvable models in statistical mechanics. *Comm. Math. Phys. 117* (1988), 243-259.

[6] Alexander, J. W. Topological invariants of knots and links. *Trans. Amer. Math. Soc. 20* (1923), 275-306.

[7] BABELON, O. Jimbo's q-analogues and current algebras. *Letters in Mathematical Physics 15* (1988), 111-117.

[8] BALL, R. and M. L. MEHTA. Sequence of invariants for knots and links. *J. Physique 42* (1981), 1193-1199.

[9] BAUER, W. R., F. H. C. CRICK and J. H. WHITE. Supercoiled DNA. *Sci. Amer. 243* (1980), 118-133.

[10] BAXTER, R. J. *Exactly Solved Models in Statistical Mechanics.* Academic Press (1982).

[11] —— On Zamolodchikov's solution of the tetrahedron equations. *Comm. Math. Phys. 88* (1983), 185-205.

[12] BIRMAN, J. S. *Braids, links and mapping class groups.* Annals of Math. Studies No. 82. Princeton University Press. Princeton, N. J. (1976).

[13] BRANDT, R. D., W. B. R. LICKORISH and K. C. MILLETT. A polynomial invariant for unoriented knots and links. *Invent. Math. 84* (1986), 563-573.

[14] BURDE, G. and H. ZIESCHANG. De Gruyter (1986).

[15] CHEW, G. F. and V. POENARU. Single-surface basis for topological particle theory. *Physical Rev. D, Vol. 32, No. 10,* Nov. 1985.

[16] CONWAY, J. H. An enumeration of knots and links and some of their algebraic properties. *Computational Problems in Abstract Algebra.* Pergamon Press, New York (1970), 329-358.

[17] CROWELL, R. H. and R. H. FOX. *Introduction to Knot Theory.* Blaisdell Pub. Co. (1963).

[18] DRINFELD, V. G. Quantum Groups. *Proc. Intl. Congress Math., Berkeley, Calif. USA* (1986), 789-820.

[19] —— Hamiltonian structures on Lie groups, Lie bialgebras and the geometric meaning of the classical Yang-Baxter equations. *Soviet Math, Dokl. 27* (1983), No. 1.

[20] —— Hopf algebras and the quantum Yang-Baxter equation. *Soviet Math. Dokl. 32* (1985), No. 1.

[21] ELLIOT, G., M. CHOI and N. YUI. Gauss polynomials and the rotation algebra. (Preprint 1988).

[22] FADDEEV, L. D., N. Yu. RESHETIKHIN and L. A. TAKHTAJAN. Quantization of Lie groups and Lie algebras. LOMI Preprint E-14-87, Steklov Mathematical Institute, Leningrad, USSR.

[23] FOX, R. H. and J. W. MILNOR. Singularities of 2-spheres in 4-space and cobordism of knots. *Osaka J. Math. 3* (1966), 257-267.

[24] FREYD, P., D. YETTER, J. HOSTE, W. B. R. LICKORISH, K. C. MILLETT and A. OCNEANU [HOMFLY]. A new polynomial invariant of knots and links. *Bull. Amer. Math. Soc. 12* (1985), 239-246.

[25] FRÖHLICH, J. Statistics of fields, the Yang-Baxter equation and the theory of knots and links. (Preprint 1987).

[26] FRÖHLICH, J. and P. MARCHETTI. Quantum field theories of vortices and anyons. (Preprint 1988).

[27] FULLER, F. B. Decomposition of the linking number of a closed ribbon: a problem from molecular biology. *Proc. Natl. Acad. Sci. USA 75* (1978), 3557.

[28] GLASSER, M. L. The free energy of the three-dimensional Zamolodchikov model. *J. Math. Phys. 27* (11), Nov. 1986.

[29] DE LA HARPE, P., M. KERVAIRE and C. WEBER. On the Jones polynomial. *L'Enseign. Math. 32* (1986), 271-335.

[30] HARTLEY, R. Conway potential functions for links. *Comment. Math. Helv. 58* (1983), 365-378.

[31] Ho, C. F. A new polynomial invariant for knots and links — preliminary report. *AMS Abstracts, Vol. 6, No. 4, Issue 39* (1985), 300.

[32] Hoste, J. A polynomial invariant of knots and links. *Pacific J. Math. 124* (1986), 295-320.

[33] Jaeger, F. A combinatorial model for the Homfly polynomial. (Preprint 1988).

[34] —— A Homfly model for the Kauffman polynomial. (Research announcement 1988).

[35] Jimbo, M. A q-difference analogue of $U(q)$ and the Yang-Baxter equation. *Lect. in Math. Physics 10* (1985), 63-69.

[36] —— Quantum R-matrix for the generalized Toda system. *Comm. Math. Phys. 102* (1986), 537-547.

[37] Jones, V. F. R. A new knot polynomial and von Neumann algebras. *Notic*es of AMS 33 (1986), 219-225.

[38] —— A polynomial invariant for links via von Neumann algebras. *Bull. Amer. Math. Soc. 12* (1985), 103-112.

[39] —— Hecke algebra representations of braid groups and link polynomials. *Ann. of Math. 126* (1987), 335-388.

[40] —— On knot invariants related to some statistical mechanics models. *Pacific J. Math. 137* (1989), 311-334.

[41] Kauffman, L. H. The Conway polynomial. *Topology 20* (1980), 101-108.

[42] —— *Formal Knot Theory.* Princeton University Press Mathematical Notes 30 (1983).

[43] —— *On Knots.* Annals of Mathematics Study 115, Princeton University Press (1987).

[44] —— State models and the Jones polynomial. *Topology 26* (1987), 395-407.

[45] —— In*variants of graphs in three-space. (To appear in Trans. Amer. Math. Soc.).*

[46] —— New invariants in the theory of knots. (Lectures given in Rome, June 1986. To appear in *Astérisque*).

[47] —— New invariants in the theory of knots. *Amer. Math. Monthly Vol. 95, No. 3*, March 1988, 195-242.

[48] —— An invariant of regular isotopy. (Announcement 1985).

[49] —— An invariant of regular isotopy. (To appear in *Trans. Amer. Math. Soc.*).

[50] —— *Knots and Physics.* (Book in preparation — based on lectures given at Universita di Bologna and Politecnico di Torino — 1985 and subsequent developments).

[51] —— Statistical mechanics and the Jones polynomial. Proceedings of the 1986 Santa Cruz conference on Artin's Braid Group. *AMS Contemp. Math. Series 78* (1988), 263-297.

[52] —— A Tutte polynomial for signed graphs. *Discrete Applied Math. 25* (1989), 105-127.

[53] —— Map coloring and the vector cross product. (To appear in *Journal of Combinatorial Theory*).

[54] —— *Map Reformulation.* Princelet Editions (1987).

[55] —— State models for knot polynomials — an introduction. (To appear in the proceedings of the meeting of the Brasilian Mathematical Society — July 1987).

[56] Kauffman, L. H. and P. Vogel. Link polynomials and a graphical calculus. (To appear).

[57] —— Kauffman, L. H. Statistical mechanics and the Alexander polynomial. *AMS Contemp. Math. Series 96* (1989), 221-231.

[58] —— Knots, abstract tensors and the Yang-Baxter equations. In *Knots, Topology and Quantum Field Theories* (ed. by L. Lusanna). World Sci. (1990), 179-334.

[59] KAUFFMAN, L. H. and S. LINS. Decomposition of the vertex group for three-manifolds. (To appear).
[60] KAUFFMAN, L. H. and S. LINS. Surface states and the Jones polynomial. (In preparation).
[61] KIRKMAN, T. P. The enumeration, description and construction of knots with fewer 10 crossings. *Trans. Royal Soc. Edin. 32* (1865), 281-309.
[62] KOHNO, T. Monodromy representations of braid groups and Yang-Baxter equations. *Ann. Inst. Fourier, Grenoble 37, 4* (1987), 139-160.
[63] KULISH, P. P., N. Yu. RESHETIKHIN and E. K. SKLYANIN. Yang-Baxter equation and representation theory: I. *Letters in Math. Physics 5* (1981), 393-403.
[64] KUNIBA, A., Y. AKUTSU and M. WADATI. Virasoro algebra, von Neumann algebra and critical eight-vertex SOS models. *J. Phys. Soc. Japan 55, No. 10* (1986), 3285-3288.
[65] LAWRENCE, R. A universal link invariant using quantum groups. (Preprint 1988).
[66] LICKORISH, W. B. R. A relationship between link polynomials. *Math. Proc. Camb. Phil. Soc. 100* (1986), 109-112.
[67] —— Linear skein theory and link polynomials. *Topology and its Applications 27* (1987), 265-274.
[68] LICKORISH, W. B. R. and K. C. MILLETT. A polynomial invariant for oriented links. *Topology 26* (1987), 107-141.
[69] LIPSON, A. S. An evaluation of a link polynomial. *Math. Proc. Camb. Phil. Soc. 100* (1986), 361-364.
[70] —— Some more states models for link invariants. *Pacific J. Math.* (To appear).
[71] MAGID, S. Non-commutative A/G: a new approach to quantization of photons. (Preprint 1988).
[72] MANIN, Yu. I. Quantum groups and non-commutative geometry. (Lecture Notes, Montreal, Canada, July 1988).
[73] —— Some remarks on Kozul algebras and Quantum groups. *Ann. Inst. Fourier., Grenoble 37, 4* (1987), 191-205.
[74] MILLETT, K. and D. JONISH. Invariants of graphs. (To appear).
[75] MORTON, H. R. Seifert circles and knot polynomials. *Math. Proc. Camb. Phil. Soc. 99* (1986), 107-109.
[76] MURASUGI, K. On the genus of the alternating knot I, II. *J. Math. Soc. Japan 10* (1958), 94-105, 235-248.
[77] —— Jones polynomials and classical conjectures in knot theory. *Topology 26* (1987), 187-194.
[78] PENROSE, R. Applications of negative dimensional tensors. *Combinatorial Mathematics and its Applications*, Edited by D. J. A. Welsh. Academic Press (1971).
[79] PRZYTYCKI, J. Skein modules of 3-manifolds. (To appear).
[80] REIDEMEISTER, K. *Knothentheorie.* Chelsea Publishing Co., New York (1948). Copyright 1932. Julius Springer, Berlin.
[81] RESHETIKHIN, N. Y. Quantized universal enveloping algebras, the Yang-Baxter equation and invariants of links, I and II. LOMI reprints E-4-87 and E-17-87, Steklov Institute, Leningrad, USSR.
[82] ROLFSEN, D. *Knots and Links.* Publish or Perish Press (1976).
[83] ROVELLI, C. and L. SMOLIN. Knot theory and quantum gravity. (Preprint 1988).
[84] SCHULTZ, C. Solvable q-state models in lattice statistics and quantum field theory. *Phys. Rev. Lettrs. 46, No. 10*, March 1981.
[85] SIMON, J. Topological chirality of certain molecules. *Topology, Vol. 25, No. 2* (1986), 229-235.
[86] TAIT, P. G. On Knots I, II, III. *Scientific Papers Vol. I*, Cambridge University Press, London, 1898, 273-347.

[87] THISTLETHWAITE, M. B. Knot tabulations and related topics. *Aspects of Topology.* Ed. I. M. James and E. H. Kronheimer. Cambridge University Press (1985), 1-76.
[88] —— A spanning tree expansion of the Jones polynomial. *Topology 26* (1987), 297-309.
[89] —— Kauffman's polynomial and alternating links. *Topology 27* (1988), 311-318.
[90] —— On the Kauffman polynomial for an adequate link. *Inventiones Math. Vol. 93, Fasc. 2* (1988), 285-296.
[91] TRACE, B. On the Reidemeister moves of a classical knot. *Proc. Amer. Math. Soc. Vol. 89, No. 4* (1983), 722-724.
[92] TRUONG, T. T. Some novel aspects of Bethe-Ansatz methods for vertex systems. *Physica 124 A* (1984), 603-612.
[93] TURAEV, V. G. The Yang-Baxter equations and invariants of links. LOMI preprint E-3-87, Steklov Institute, Leningrad, USSR. *Inventiones Math.* (1988).
[94] —— A simple proof of Murasugi and Kauffman theorems on alternating links. *L'Enseign. Math. 33* (1987), 203-225.
[95] —— The Conway and Kauffman modules of the solid torus with an appendix on the operator invariants of tangles. LOMI preprint E-6-88, Steklov Mathematical Institute, Leningrad, USSR.
[96] TZE, C. H. Manifold-splitting regularization, self-linking, twisting, writhing numbers of spacetime ribbons and Polyakov's Fermi-Bose transmutations. (Preprint 1988).
[97] WHITE, J. H. Self-linking and the Gauss integral in higher dimensions. *Amer. J. Math. Vol. XCI, July* (1969), 693-728.
[98] WHITNEY, H. On regular closed curves in the plane. *Comp. Math. 4* (1937), 276-284.
[99] WITTEN, E. Lecture. *Intl. Congress on Math. Physics*, Swansea, Wales, July 1988.
[100] ZAMOLODCHIKOV, A. B. Factorized *S*-matrices and lattice statistical systems. *Soviet Sci. Reviews. Part A* (1979-1980).
[101] —— Tetrahedron equations and the relativistic *S*-matrix of straight strings in 2 + 1 dimensions. *Comm. Math. Phys. 79* (1981), 489-505.

(Reçu le 30 octobre 1988)

Louis H. Kauffman

Department of Mathematics, Statistics and Computer Science
University of Illinois at Chicago, Box 4348
Chicago, Illinois 60680 (USA)

and

Institut des Hautes Études Scientifiques
35, route des Chartres
F-91440, Bures-sur-Yvette (France)

vide-leer-empty