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## THE POMPEIU PROBLEM REVISITED

by S. C. BAGCHI and A. SITARAM

ABSTRACT. One of the central results connected with the Pompeiu problem is a theorem of Brown, Schreiber and Taylor. Using some old work of the authors on spectral synthesis, a proof of this result is given. Though separately dealt with, it is shown that some of the main results for the Pompeiu problem for non-Euclidean symmetric spaces can also be treated in the same spirit. In all the cases the role of representations of the underlying group of isometries is highlighted. This point of view leads to some new results for the Pompeiu problem for two sided translations on the non-commutative groups  $SL(2, \mathbf{R})$  and  $M(2)$ . Finally, a brief discussion is provided for some related problems.

### 1. INTRODUCTION

Let  $X$  be a locally compact Hausdorff space and  $G$  a group of homeomorphisms of  $X$  each of which leaves a given non-negative Radon measure  $\mu$  invariant. The central theme of this article is what is known in the literature as the Pompeiu property: a relatively compact measurable subset  $E \subseteq X$  is said to have the Pompeiu property if for a continuous function  $f$  on  $X$ ,

$$(1.1) \quad \int_{gE} f(x) d\mu(x) = 0 \quad \text{for all } g \in G$$

implies  $f \equiv 0$ .

The Pompeiu property in a wide variety of settings and its relation to other problems have been the subject-matter of a large number of investigations beginning with two articles by the Roumanian mathematician D. Pompeiu in 1929 ([18], [19]). In the first paper ([18]) the set-up was essentially  $X = \mathbf{R}^2$  with the Lebesgue measure  $\mu$  and  $G$  the group  $\mathbf{R}^2$