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## LINK SIGNATURE, GOERITZ MATRICES AND POLYNOMIAL INVARIANTS

by A. S. LIPSON

### ABSTRACT

Lickorish and Millett introduced the notion of skein equivalence of oriented links in [15]. In the first part of this paper I show that the link signature function  $\sigma_L: S^1 \rightarrow Z$  of [22], [12], etc. is a skein invariant for links with non-zero Alexander polynomial. In the second part I show that a renormalised form of Kauffman's polynomial invariant  $F_L(a, z)$ , well-defined on non-oriented links is calculable from the Goeritz matrix.

### 1. *P*-SKEINS AND SIGNATURE

In this section I present two notions of "skein equivalence" for links; "broad" skein equivalence and "narrow" skein equivalence. Narrow skein equivalence is a stronger relation (i.e. has smaller equivalence classes), but it is not clear whether it is strictly stronger. I show that the link signature function  $\sigma_L: S^1 \rightarrow Z$  is a broad skein invariant for all links with non-zero Alexander polynomial. It is not known whether this result extends to links with zero Alexander polynomial, but it seems unlikely that it should.

#### 1.1. PRELIMINARIES

I briefly recap on some standard material. See [19], [2] or [5] for further details. Let  $L$  be an oriented link. Then it is always possible to find an oriented surface  $F$  embedded in  $S^3$  in such a way that  $\partial F = L$ , with the appropriate orientation. Such a surface is called a *Seifert surface* for the link  $L$ . Now let  $c_1, \dots, c_n$  be closed curves lying in  $S$  whose homology classes generate  $H_1(S)$ , and let  $c_1^+, \dots, c_n^+$  be the results of pushing these curves slightly away from  $S$  in the positive direction in a collar neighbourhood of the surface. The matrix  $V = (v_{ij})$ , where  $v_{ij} = lk(a_i, a_j^+)$ , is