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LINK SIGNATURE, GOERITZ MATRICES AND POLYNOMIAL INVARIANTS

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ABSTRACT

Lickorish and Millett introduced the notion of skein equivalence of oriented links in [15]. In the first part of this paper I show that the link signature function $\sigma_L: S^1 \to Z$ of [22], [12], etc. is a skein invariant for links with non-zero Alexander polynomial. In the second part I show that a renormalised form of Kauffman's polynomial invariant $F_L(a, z)$, well-defined on non-oriented links is calculable from the Goeritz matrix.

1. *P*-SKEINS AND SIGNATURE

In this section I present two notions of "skein equivalence" for links; "broad" skein equivalence and "narrow" skein equivalence. Narrow skein equivalence is a stronger relation (i.e. has smaller equivalence classes), but it is not clear whether it is strictly stronger. I show that the link signature function $\sigma_L: S^1 \to Z$ is a broad skein invariant for all links with non-zero Alexander polynomial. It is not known whether this result extends to links with zero Alexander polynomial, but it seems unlikely that it should.

1.1. PRELIMINARIES

I briefly recap on some standard material. See [19], [2] or [5] for further details. Let L be an oriented link. Then it is always possible to find an oriented surface F embedded in S^3 in such a way that $\partial F = L$, with the appropriate orientation. Such a surface is called a *Seifert surface* for the link L. Now let $c_1, ..., c_n$ be closed curves lying in S whose homology classes generate $H_1(S)$, and let $c_1^+, ..., c_n^+$ be the results of pushing these curves slightly away from S in the positive direction in a collar neighbourhood of the surface. The matrix $V = (v_{ij})$, where $v_{ij} = lk(a_i, a_j^+)$, is