

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 37 (1991)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE EVALUATION OF SELBERG CHARACTER SUMS
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Kurzfassung

DOI: <https://doi.org/10.5169/seals-58741>

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THE EVALUATION OF SELBERG CHARACTER SUMS

by Ronald J. EVANS

ABSTRACT. The evaluations of Selberg character sums conjectured on p. 207 of *Enseignement Math.* 27 (1981) are proved.

§1. INTRODUCTION

Many of the classical special functions over \mathbf{C} have character sum analogs over finite fields. For example, the Gauss and Jacobi sums defined in (1.1) are analogs of the gamma and beta integrals

$$\Gamma(a) = \int_0^{\infty} e^{-x} x^a \frac{dx}{x}, \quad \beta(a, b) = \int_0^1 x^a (1-x)^b \frac{dx}{x(1-x)}.$$

Some identities for character sums over finite fields seem more difficult to prove than their classical counterparts; compare, e.g., the Hasse-Davenport product formula for Gauss sums [7, (7)] with the Gauss multiplication formula for gamma functions. The identities for n -dimensional Selberg character sums given in Theorems 1.1, 1.1a provide further examples. Their counterparts are the well known n -dimensional Selberg integral extensions of the gamma and beta integral formulas.

The case $n = 3$ of the Selberg character sum identity in Theorem 1.1 has been used to evaluate a sum connected with the root system G_2 [8]. The case $n = 2$ is equivalent to an analog of Dixon's summation formula [11, (2.1.5)] involving hypergeometric ${}_3F_2$ character sums over finite fields. We remark that hypergeometric character sums have been used, e.g., in the computation of the number of points on hypersurfaces [13], [12], in proving congruences for Apéry numbers [14], and in graph theory [6], [9].

Let $GF(q)$ be a finite field of q elements, where q is a power of an odd prime. Fix a multiplicative character $\tau: GF(q)^* \rightarrow \mathbf{C}^*$ of order $q - 1$ and a nontrivial additive character $\psi: GF(q) \rightarrow \mathbf{C}^*$. Extend τ by defining $\tau(0) = 0$. Let $\phi = \tau^{(q-1)/2}$ be the quadratic character on $GF(q)$. For all integers a, b , define the Gauss sums $G(a)$ and Jacobi sums $J(a, b)$ by