Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 37 (1991)

Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE EVALUATION OF SELBERG CHARACTER SUMS

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Bibliographie

DOI: https://doi.org/10.5169/seals-58741

$$b + jc \not\equiv 0 \pmod{q-1} \quad \text{for all} \quad j \quad \text{with} \quad 0 \leqslant j \leqslant n-1 ,$$

in view of [10, Lemmas 2.1 and 2.2]. Assume also that

$$c \not\equiv 0 \pmod{q-1} ,$$

since the result has been proved in [5] for $c \equiv 0 \pmod{q-1}$.

Theorem 1.1 is clear for $n = 1$, so let $n > 1$ and assume as induction hypothesis that

$$S_{n-1}(a+c, b+c, c) = P_{n-1}(a+c, b+c, c) .$$

By (3.8) and (3.12), if $d \nmid n$,

$$\begin{aligned} S_n(a,b,c) &= P_{n-1}(a+c, b+c, c) \frac{G(a)G(b)G(cn)\bar{G}(a+b+(n-1)c)}{qG(c)} \\ &= P_n(a,b,c) , \end{aligned}$$

whereas

$$S_n(a,b,c) + (q-1)P_n(a,b,c) = qP_n(a,b,c) , \quad \text{if} \quad d \mid n .$$

Thus $S_n(a,b,c) = P_n(a,b,c)$ in both cases, proving Theorem 1.1. The proofs of Theorems 1.1*a* and 1.1*b* follow similarly, from (3.8*a*), (3.12*a*) and (3.8*b*), (3.12*b*) in place of (3.8), (3.12).

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(Reçu le 3 janvier 1991)

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