the barycenter of $\mu v_0, .., \widehat{\mu v_i}, .., \mu v_n$, which is $g_i(e_1 + e_2 + .. + e_{n-1})$, is consequently also in $\mathbf{Z}^n$.

So the barycenters of all faces of μS_v are in $\mathbf{Z}^n$ and they are the vertices of an integral simplex S'.

Calculating the first coordinate of the barycenter of $\widehat{\mu v_0}, \mu v_1, .., \mu v_n$ we see that n divides $\mu + \mu(l-1) + \mu a_1$.

Calculating the first coordinate of the barycenter of $\mu v_0, \widehat{\mu v_1}, \mu v_2, .., \mu v_n$, we see that n divides $\mu(l-1) + \mu a_1$.

So the integer n divides μ too but this implies that $\mu = n$ and hence $l = 1$. This and the affine regularity imply that S is small-faced. $\square$

The notions of affine regularity and of integrality may both be generalized to other polytopes, such as hypercubes, cross-polytopes, hexagones in dimension 2 or exceptional polytopes in dimension 4. We plan to consider these in a further paper.

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Roland Bacher

Section de mathématiques
2-4, rue du Lièvre
CP 240
CH-1211 Genève 24