Proof. Let M be a nilmanifold. Then

$$\begin{aligned}
\dim M + 1 &= e_0(M) + 1 \\
&= e_0(M \times S^k) \text{ since } e_0 \text{ respects products} \\
&\leqslant \operatorname{cat}(M \times S^k) \\
&\leqslant \operatorname{cat}(M) + 1 \text{ Fox's inequality} \\
&= \dim M + 1 .
\end{aligned}$$

Hence all inequalities are equalities and $\operatorname{cat}(M \times S^k) = \operatorname{cat}(M) + 1$. □

ADDED IN PROOF. By using the equality $e_0(M) = \dim(M)$ and extending the e_0-invariant to maps, C. McCord and the author have given a proof of the Arnold Conjecture for nilmanifolds (cf. C. McCord and J. Opera, *Rational Ljusternik-Schnirelmann Category and the Arnold Conjecture for Nilmanifolds,* preprint 1992). That is, any smooth 1-periodic Hamiltonian system on a symplectic nilmanifold M has at least $\dim(M) + 1$ contractible 1-periodic orbits.

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