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EMMY NOETHER: HIGHLIGHTS OF HER LIFE AND WORK

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Emmy Noether was a towering figure in the evolution of abstract algebra. In fact, she was the moving spirit behind the abstract, axiomatic approach to algebra. She also had a singular personality which attracted a group of students and collaborators who spread the gospel of abstract algebra far and wide. I will first give a sketch of her life and then discuss some of her work, including her intellectual debts and her legacy.

A. HER LIFE

Emmy Noether was born in 1882 in Erlangen, the German university town of Klein's Erlangen Programme fame. The university was founded in 1743 and had among its mathematics faculty such luminaries as von Staudt, Klein, Gordan (the "king of invariants"), and Max Norther, the famous algebraic geometer and Emmy's father. Gordan was a contemporary and friend of Max Noether and a frequent visitor of the Noethers. Although Emmy showed little early interest in mathematics, the frequent mathematical conversations at the Noether household between Gordan and her father were an important part of the atmosphere in which she grew up. Gordan was later to become Emmy Noether's thesis advisor.

Emmy Noether came from an economically well-established household, and her childhood seems to have been happy. She liked dancing and took piano lessons (which she did not like). She was a friendly and likeable child. Between the ages of seven and fifteen she went to the "Municipal School for Higher Education of Daughters", where she studied English and French.[1]) In 1900, at the age of eighteen, she was certified as a teacher of both subjects in

[1]) There is no indication that she wanted to study mathematics or science, but these were, of course, not "feminine" subjects.

"Institutions for the Education and Instruction of Females". She chose, however, not to pursue this career and instead enrolled at the University of Erlangen.

Easier said than done in those days. As the famous German historian Heinrich von Treitschke put it in the early 1890s ([17], p. 17):

> Many sensible men these days are talking about surrendering our universities to the invasion of women, and thereby falsifying their entire character. This is a shameful display of moral weakness. They are only giving way to the noisy demands of the Press. The intellectual weakness of their position is unbelievable... The universities are surely more than mere institutions for teaching science and scholarship. The small universities offer the students a comradeship which in the freedom of its nature is of inestimable value for the building of a young man's character...

In 1898, two years before Emmy Noether entered the University of Erlangen, the Senate of the University declared that the admission of women students would "overthrow all academic order" ([24], p. 10). By 1900, however, the authorities relented and extended to women the *conditional* right to enrol in German universities.[1]) Individual professors had the right, which they often exercised, to deny women permission to attend their lectures. This meant that Emmy Noether (one of two women among 1,000 students at the university) had to choose her subjects and instructors with some care. In fact, she at first took courses in history and modern languages, but later she switched to mathematics — it is not clear exactly when and why. By 1904 she was *formally* able to register as a student at the University of Erlangen, now studying only mathematics. In 1908 she received her Ph. D. degree, summa cum laude, having written a thesis on invariants under Gordan.

Between 1908 and 1915 Emmy Noether worked *without compensation* at the University of Erlangen. "Working" meant doing research, attending meetings of the German Mathematical Society and giving presentations, and occasionally substituting at lectures for her ailing father.

Although Emmy Noether did not have a formal position during these seven years, they were, as noted, not idly spent; and they bore fruit. She had become an expert on invariant theory, to the point that in 1915 Hilbert and Klein invited her to Göttingen to help them with problems on differential invariants.

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[1]) Women were permitted to enrol at universities in the U.S. in 1853, in France in 1861, in England in 1878, and in Italy in 1885.

These proved important in connection with their work on mathematical aspects of the general theory of relativity.[1])

Emmy Noether's move to Göttingen was of singular importance. Göttingen was at that time considered the world center of mathematics. With Gauss, Dirichlet, and Riemann as former professors, and with the contemporary faculty including Klein, Hilbert, Landau, Minkowski, and Courant, and later Weyl, Bernays, and Neugebauer, Göttingen had become the "Mecca of Mathematics". The list of visitors reads like a "who's who" of the world of mathematics: van der Waerden from Holland, Olga Taussky and Köthe from Austria, Tagaki and Shoda from Japan, André Weil and Chevalley from France, O. Schmidt, Gelfond, Alexandrov, Kolmogorov, and Urysohn from the Soviet Union, Tsen from China, Kuratowski from Poland, MacLane, G. D. Birkhoff, Wiener, and Lefschetz from the United States, and Artin, Hasse, Brauer, Siegel, and von Neumann from various universities in Germany.

Emmy Noether thrived in these surroundings. The decade 1920-1930 was the decisive period of her mathematical life. This is when she made her groundbreaking contributions to algebra. (She was then in her forties[2])) She attracted students, co-workers, and visitors who vigorously developed the subject soon to become known as "modern algebra". About 1930, the algebraists around Noether had gained the reputation as the most active group at the Mathematical Institute of Göttingen — quite an accomplishment given the presence of the likes of Hilbert, Weyl, Landau, and Courant! Two great honours came her way in 1932. First, she was awarded, jointly with Artin, the "Ackermann-Teubner Memorial Prize" for the advancement of the mathematical sciences. Second, she gave one of the 21 plenary addresses at the International Congress of Mathematicians in Zurich.[3])

But all did not go smoothly for Emmy Noether at Göttingen. One began a university career in Germany as Privatdozent (comparable in rank to an assistant professor). This was an *unpaid* position which gave its holder the right to teach.[4]). The income of Privatdozenten consisted of minimal fees paid by students for attending their lectures. One would have thought that under

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[1]) It was 1915, and Einstein had just promulgated his general theory of relativity. Both Hilbert and Klein turned their attention to it.

[2]) "Such a late maturing is a rare phenomenon in mathematics", notes Weyl ([41], p. 128), mentioning Sophus Lie as another great exception to the rule.

[3]) It was a remarkable event for a woman to be invited to give a plenary talk.

[4]) The Privatdozent, unlike the professor, was not an appointee of the state and hence received no salary.

such circumstances, and having been invited to Göttingen by the great Klein and Hilbert, Emmy Noether would have got an appointment as Privatdozent immediately upon her arrival in Göttingen. That was not to be, however. The philologists and historians of the Philosophical Faculty (of which mathematics was part) opposed Hilbert's efforts to allow Emmy to habilitate (a necessary step in becoming Privatdozent), because she was a woman. Hilbert protested, without success, to the University Senate: "After all", he claimed, "we are a university, and not a bathing establishment" ([41], p. 125). Only four years later (in 1919) was Noether allowed to habilitate and become Privatdozent. This followed the war, which brought profound political and social change, including an improvement in the legal position of women.

Three years later, in 1922, the mathematics department of Göttingen applied to the Ministry of Education to appoint Noether as professor. She was given the title "extraordinary professor without tenure" ("extraordinary" is the equivalent of an associate professor). This was merely a title, carrying no obligations and no salary. Since the high postwar inflation in Germany greatly reduced students' ability to pay their instructors, Noether was fortunate to get in the following year a "Teaching Assignment" ("Lehrauftrag") in Algebra, which provided a small remuneration. It required, however, annual confirmation by the Ministry. This is the position she remained in until she left Göttingen ten years later.

Why was there little *institutional* recognition of Emmy Noether's talents and accomplishments? We can only speculate, of course. But she had several marks against her: she was a woman, she was Jewish, and she had leftist political sympathies.

What kind of teacher was Emmy Noether? By standard measures, she was not a good teacher. She did not give well-organized, polished lectures. Yet, she inspired many students, through her lectures and through personal contact. Here is testimony from some who attended her courses:

> She was concerned with concepts only, not with visualization or calculation... This... was probably one of the main reasons why her lectures were difficult to follow... And yet, how profound the impact of her lecturing was! (Van der Waerden [37], p. 110).
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> Professor Noether's lectures... are... excellent, both in themselves and because they bear an entirely different character in their excellence. Professor Noether thinks fast and talks faster. As one listens, one must also think fast — and that is always excellent training. (MacLane [29], p. 77).
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> To an outsider Emmy Noether seemed to lecture poorly, in a rapid and confusing manner, but her lectures contained a tremendous force of

mathematical thought and an extraordinary warmth and enthusiasm. (Alexandrov [2], p. 165).

Indeed, Emmy Noether had a warm and caring personality. She was also modest, generous, frank, strong-willed, and outwardly coarse. "She was both a loyal friend and a severe critic", said van der Waerden ([37], p. 111), giving expression to one of these seeming contradictions. Her personal traits, combined with deep mathematical insights, attracted a core of devoted students, the so-called "Noether boys".[1]) They often visited her home, and they used to go on frequent walks together. The topic of conversation was almost invariably mathematics. Here is the story of one such walk.

It was raining, and Emmy Noether's umbrella did not offer much protection since it was in poor condition. When her students suggested that she get it repaired, she replied: "Quite right, but it can't be done: when it doesn't rain, I don't think of the umbrella, and when it rains, I need it" ([12], p. 48).

In a more serious vein, van der Waerden relates the following ([38], p. 173):

> I wrote a paper based upon this simple idea and showed it to Emmy Noether. She at once accepted it for the *Mathematische Annalen,* without telling me that she had presented the same idea in a course of lectures just before I came to Göttingen. I heard it later from Grell, who had attended her course.[2])

On January 31, 1933 Hitler assumed the office of Chancellor. On March 31 he announced the beginning of the Third Reich. On April 25 Emmy Noether was dismissed from her teaching position. The dismissal of Courant, Landau, and Bernays followed in short order. Courant was replaced as head of the Mathematics Institute at Göttingen by Neugebauer, who lasted one day in that position. He refused to sign the required loyalty declaration.

With Weyl's assistance, Emmy Noether got a visiting position at Bryn Mawr College in Pennsylvania. The transition might have been difficult but for the warm reception she received at Bryn Mawr and the mathematical contacts she established at nearby Princeton. At Bryn Mawr she had her "Noether girls" — one doctoral and three postdoctoral students (among the latter was Olga Taussky). At Princeton she began (in early 1934) to give weekly lectures on algebra. Writing to Hasse about them, she said: "I'm beginning to realize that I must be careful; after all, they are essentially used to explicit

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[1]) Among her Ph.D. students were Deuring, Fitting, Grell, Greta Hermann, Krull, Levitzki, F.K. Schmidt, Ruth Stauffer, and Witt.

[2]) Emmy Noether was a collaborator in the editing of *Mathematische Annalen,* but she was hurt that this work was never explicitly recognized. Grell was one of her Ph.D. students.

computation and I have already driven a few of them away with my approach" ([12], pp. 81-82). Among those who were not driven away were Albert, Brauer, Jacobson, Vandiver, and Zariski. In a recent book on Zariski, Carol Parikh pointed out that "Zariski's contact with Noether was undoubtedly the single most important aspect of that year for him" ([33], p. 74).

The time she spent at Bryn Mawr and Princeton was the happiest in her life, Emmy Noether told Veblen before her death. She was respected and appreciated as she had never been in her own country. But it was a brief, if happy, year and a half. On April 10, 1935 she underwent an operation for a tumor. She was recovering well when, four days later, complications brought unexpected death.

Ten days after her death Hermann Weyl delivered at Bryn Mawr a moving and eloquent eulogy. Let me conclude this account of Emmy Noether's life by quoting from it ([41], pp. 132, 149-152; for further details about her life see [7], [12], [24], and [36]):

> It was only too easy for those who met her for the first time, or had no feeling for her creative power, to consider her queer and to make fun at her expense. She was heavy of build and loud of voice, and it was often not easy for one to get the floor in competition with her. She preached mightily, and not as the scribes. She was a rough and simple soul, but her heart was in the right place. Her frankness was never offensive in the least degree. In everyday life she was most unassuming and utterly unselfish; she had a kind and friendly nature. Nevertheless she enjoyed the recognition paid her; she could answer with a bashful smile like a young girl to whom one had whispered a compliment. No one could contend that the Graces had stood by her cradle; but if we in Göttingen often chaffingly referred to her as "der Noether" (with the masculine article), it was also done with a respectful recognition of her power as a creative thinker who seemed to have broken through the barrier of sex. She possessed a rare humor and a sense of sociability; a tea in her apartment could be most pleasurable... She was a kind-hearted and courageous being, ready to help, and capable of the deepest loyalty and affection. And of all I have known, she was certainly one of the happiest...
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> Two traits determined above all her nature: First, the native productive power of her mathematical genius. She was not clay, pressed by the artistic hands of God into a harmonious form, but rather a chunk of human primary rock into which he had blown his creative breath of life. Second, her heart knew no malice; she did not believe in evil – indeed it never entered her mind that it could play a role among men. This was never more

> forcefully apparent to me than in the last stormy summer, that of 1933, which we spent together in Göttingen... A time of struggle like this one... draws people closer together; thus I have a particularly vivid recollection of these months. Emmy Noether, her courage, her frankness, her unconcern about her own fate, her conciliatory spirit, were in the midst of all the hatred and meanness, despair and sorrow surrounding us, a moral solace... The memory of her work in science and of her personality among her fellows will not soon pass away. She was a great mathematician, the greatest, I firmly believe, that her sex has ever produced, and a great woman.

B. Her work

I will now give an account of some of Emmy Noether's major contributions to mathematics, indicating their sources.

Irving Kaplansky called her the "mother of modern algebra" ([23], p. 155). Saunders MacLane asserted that "abstract algebra, as a conscious discipline, starts with Emmy Noether's 1921 paper 'Ideal Theory in Rings'" ([28], p. 10). Hermann Weyl claimed that she "changed the face of algebra by her work" ([41], p. 128). It is a tall order to try to do justice to these assertions, but let me try.

According to van der Waerden, the essence of Emmy Noether's mathematical credo is contained in the following maxim ([5], p. 42):

> All relations between numbers, functions and operations become perspicuous, capable of generalization, and truly fruitful after being detached from specific examples, and traced back to conceptual connections.

We identify these ideas with the abstract, axiomatic approach in mathematics. They sound commonplace to us. But they were not so in Emmy Noether's time. In fact, they are commonplace today in considerable part *because* of her work.

Algebra in the 19th century was concrete by our standards. It was connected in one way or another with real or complex numbers. For example, some of the great contributors to algebra in the 19th century, mathematicians whose works shaped the algebra of the 20th century, were Gauss, Galois, Jordan, Kronecker, Dedekind, and Hilbert. Their algebraic works dealt with quadratic forms, cyclotomy, field extensions, permutation groups, ideals in rings of integers of algebraic number fields, and invariant theory. All of these works were related in one way or another to real or complex numbers.