

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 40 (1994)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** THE PROUHET-TARRY-ESCOTT PROBLEM REVISITED  
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**Kapitel:** 6. Open Problems  
**DOI:** <https://doi.org/10.5169/seals-61102>

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## 6. OPEN PROBLEMS

There are many open questions and unproven conjectures about the Prouhet-Tarry-Escott problem. We conclude by listing a few.

1. Find an ideal solution for any size higher than 10 or find some degree for which an ideal solution does not exist. (Even a heuristic argument would be of interest.)
2. Find another class of solutions of size 9 or 10.
3. Prove  $N(k) \leq o(k^2)$ .
4. Prove  $M(k) \leq O(k^2)$ .
5. Show that there is no 7 factor (degree 6) pure product of norm 14.
6. Find a non-trivial lower bound for  $A(k)$ . Almost equivalently prove

$$\min_{n_1, \dots, n_k} \left\| \prod_{i=1}^k (1 - x^{n_i}) \right\|_1 > 2k$$

for some  $k$ . (Problem 5 is the  $k = 7$  case of this.)

7. Find a true algorithm, even an impractical one, that determines if there is an ideal solution of size 11.
8. Find a true algorithm, even an impractical one, that determines if there is a degree 6 ( $k = 7$ ) pure product of norm 14.
9. Solve the ideal problem mod  $p^n$  for all primes  $p$  smaller than the size of the solution and all  $n$ .

The big prize is to find ideal solutions of all degrees, if indeed they exist. Question 1 above is, of course, the first step. No progress on questions 3 and 4 has been made for many years. Questions 5, 6, and 8 all relate to the Erdős-Szekeres Problem. The issue in Questions 7 and 8 is that it is not known how to bound solutions so as to make the problems finite. Question 9 is raised in [17] and would show that no local obstructions exist to solutions.

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