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Artikel: ON THE GAUSS-BONNET FORMULA FOR LOCALLY SYMMETRIC SPACES OF NONCOMPACT TYPE

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Proof. By Proposition 2.2 there is an exhaustion $V = \bigcup_{s \geq 0} V(s)$ of V by Riemannian polyhedra $V(s)$. Each polyhedron $V(s)$ in this exhaustion is equipped with the Riemannian metric induced by the one of V. Proposition 1.1 applied to $V(s)$ yields

$$\left| (-1)^n \chi'\big(V(s)\big) - \int_{V(s)} \Psi \, dv \right| \prec \sum_{k=1}^{q} \sum_{E} \int_{V_E^{n-k}(s)} \int_{O(p)} \|\Psi_{E,k}\| \, d\omega_{k-1} \, dv_E(p)$$

where $q = \dim A$ is the $\mathbb{Q}$-rank of $\mathbf{G}$ (see Section 2.1) and where the index E runs through a finite set. As we remarked in Section 1 the function $\Psi_{E,k}$ is locally computable from the components of the metric and the curvature tensor of $V(s)$ and from the components of the second fundamental form of $V_E^{n-k}(s)$ in $V(s)$. The fact that V is locally symmetric together with Lemma 3.2 thus implies that $\|\Psi_{E,k}\| \prec 1$ for all E, k. Using Lemma 3.4 we conclude that

$$\left| (-1)^n \chi'\big(V(s)\big) - \int_{V(s)} \Psi dv \right| \prec \sum_{k,E} \operatorname{Vol}\big(V_E^{n-k}(s)\big) \prec e^{-cs} \sum_{k=1}^{q} s^{q-k}.$$

By Proposition 2.3 we have $\chi'\big(V(s)\big) = \chi(V)$. The polyhedra $V(s)$ exhaust V and $\chi(V)$ is an integer; hence $(-1)^n \chi(V) = \int_{V(s)} \Psi \, dv$ for sufficiently large s. Finally, for n odd $\Psi \equiv 0$ by definition (see [AW]) and the claimed formula follows. □

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