

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 43 (1997)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON CYCLOTOMIC POLYNOMIALS, POWER RESIDUES, AND RECIPROCITY LAWS
Autor: Sharifi, Romyar T.
Kurzfassung
DOI: <https://doi.org/10.5169/seals-63283>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 01.04.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

ON CYCLOTOMIC POLYNOMIALS,
POWER RESIDUES, AND RECIPROCITY LAWS

by Romyar T. SHARIFI

ABSTRACT. For a positive integer n , let $\Phi_n(X)$ be the n th cyclotomic polynomial over the rationals, i.e., the monic irreducible polynomial which has as its roots the primitive n th roots of unity. Fix an odd prime q and let s be the largest integer such that q^s divides n . If p is a prime of the form $p = \Phi_n(qx)$ for some integer x , then all integers dividing x are q^s th powers modulo p . An analogous statement holds for the case $q = 2$. The proofs make use of norm residue symbols in cyclotomic extensions of the q -adic rationals.

1. INTRODUCTION

This paper is concerned with an interesting property of power residues of primes which appear as values of a cyclotomic polynomial. To gain an understanding of power residues, we could start by looking for patterns in a list of primes and the index of various integers modulo these primes. The case of quadratic residues is well-known, dating back to Euler, Legendre, and Gauss. We might notice, for instance, that a number a is a quadratic residue modulo primes of the form $4x + 1$, where x is a multiple of a . In general, those primes which have a given number a as a quadratic residue are completely determinable using the law of quadratic reciprocity. Indeed, this problem was one of the main motivations for the formulation of this law.

As an attempt to extend the quadratic case, we can look for a polynomial that produces primes which have a as a cubic residue. In doing so, we may discover that a is a cubic residue of primes of the form $9x^2 + 3x + 1$, where x is a multiple of a . A complete classification of cubic residues is