

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 43 (1997)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: EVEN NON-SPIN MANIFOLDS, SPIN_c STRUCTURES, AND DUALITY
Autor: ACOSTA, Daniel / LAWSON, Terry
Kurzfassung
DOI: <https://doi.org/10.5169/seals-63269>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 14.03.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

EVEN NON-SPIN MANIFOLDS,
SPIN^c STRUCTURES, AND DUALITY

by Daniel ACOSTA and Terry LAWSON

ABSTRACT. This note explores the restrictions on the second Stiefel-Whitney class w_2 of a smooth closed oriented 4-manifold which has an even intersection form but is not spin. The Hom dual of w_2 is shown to be non-integral, whereas the existence of a spin^c structure means that its Poincaré dual is the reduction of an integral class. We examine this in detail in a simple example $S^2 \times S^2 / \{\pm 1\}$.

In [H, p. 23] N. Habegger gives $M = S^2 \times S^2 / (x, y) \sim (-x, -y)$ as an example of an oriented, non-spin smooth 4-manifold with an even intersection form. In discussing this example in [K, p. 27] R. Kirby seems to be relating it to the (non-)integrality of the second Stiefel-Whitney class $w_2(M)$. However, for a closed, oriented, smooth 4-manifold X , it is always the case that the second Stiefel-Whitney class $w_2(X)$ is the mod 2 reduction of an integral class. This was first shown by Hirzebruch and Hopf in [HH, p. 169], and is a key step in showing that X admits a spin^c structure. Spin^c structures have recently become very important as they are involved in the Seiberg-Witten invariants, now a major area of study in the topology and geometry of 4-manifolds (see, e.g., [W], [T], [KM]).

In this expository paper we want to explore some of the interesting phenomena at work in this example and describe the characterizing property which w_2 possesses. On the way we shall encounter many important concepts in geometric topology, including Poincaré and Hom duality, the intersection form, even forms, spin structures, spin^c structures, and characteristic classes. The article is intended for readers who have a background of a year-long graduate course in topology from a text such as Bredon [B].

We start by reviewing some basic definitions. We will assume X is a compact, oriented smooth 4-manifold. When the coefficient group is not