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## AMENABILITY AND GROWTH OF ONE-RELATOR GROUPS

by Tullio G. CECCHERINI-SILBERSTEIN and Rostislav I. GRIGORCHUK

**ABSTRACT.** An algorithm showing whether a group given by a one-relator presentation is amenable or not is constructed. Sufficient conditions for a one-relator group of exponential growth to have uniformly exponential growth are also given.

### 0. INTRODUCTION

A one-relator group is a group  $G$  which admits a presentation

$$(*) \quad G = \langle a_1, a_2, \dots, a_m : R(a_1, a_2, \dots, a_m) = 1 \rangle$$

with one defining relation.

The paper by G. Baumslag [B 1] is a comprehensive survey of results about one-relator groups. In particular this paper stresses the role of algorithmic problems in the theory of one-relator groups.

Recently the interest in functional-analytical and asymptotical properties of one-relator groups has increased. For instance, the entropy of one-relator groups was discussed in [GrLP], random walks and Markov operators on one-relator groups were investigated in [CV], [BCCH], [BC], and the K-functor of reduced  $C^*$ -algebras of one-relator groups was computed in [BBV]. Also the growth functions of the groups  $\Gamma_n = \langle t, a : tat^{-1} = a^n \rangle$ ,  $n \neq 0, \pm 1$ , and of some other one-relator groups were calculated in [CEG] and [EJ].

Recall that a discrete group  $G$  is amenable if there exists a finitely additive measure  $\mu: \mathcal{P}(G) = \{0, 1\}^G \rightarrow [0, 1]$  which is  $G$ -(left)-invariant ( $\mu(gE) = \mu(E)$  for all  $g \in G$  and  $E \subset G$ ) and such that, in addition,  $\mu(G) = 1$ . For our purpose it will be enough to know that a group containing a free subgroup of rank two is not amenable, and that, on the contrary, any solvable group is amenable ([G]).